Tightening and Blending

Subject to Set-Theoretic Constraints

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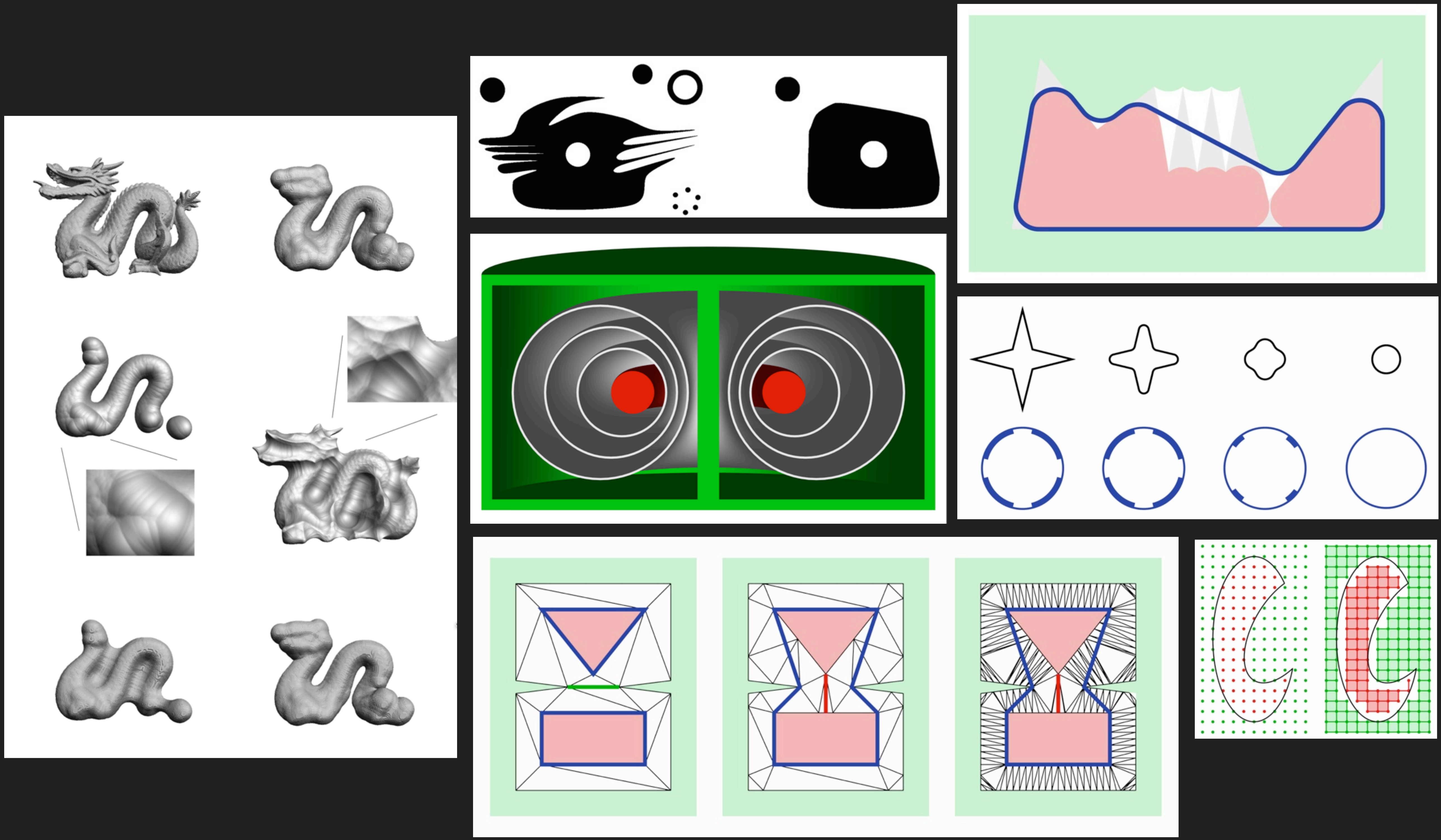
Jason Williams



PREVIEW: EXAMPLES OF TIGHTENING AND BLENDING

MASON, TIGHTENING, TIGHT HULL, TIGHT BLEND, MEDIAL COVER

Five Novel Techniques

- **Published:**
  - **Mason:** Blend while matching input shape.
  - **Tightening:** Bound curvature, minimizing boundary.
- **Unpublished:**
  - **Tight hull:** Generalize constrained convex hull.
  - **Tight blend:** Simplify tight hull normal field.
  - **Medial cover:** Obtain topology from medial axis.

Unified Problem Description

- Find an optimal surface separating two sets.
  - **Blend:** Minimize thin, sharp details.
  - **Tighten:** Minimize boundary measure.
- Solid includes one set, excludes a second.

Significant Applications

- **Model geometry:** simplification, analysis
- **Manufacture:** boundary optimization
- **CAD/biomedical image volume:** reconstruction, convergent boundary properties and normals
- **Polygonal models:** error repair

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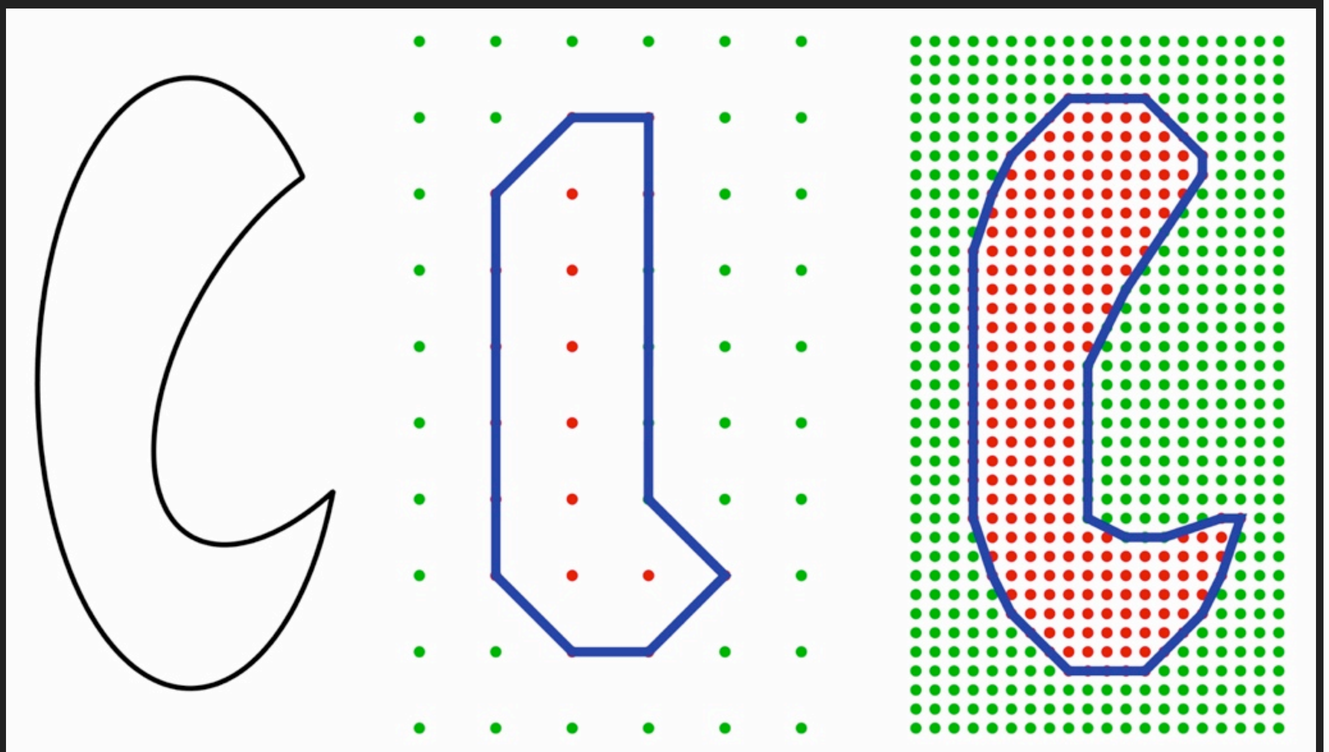
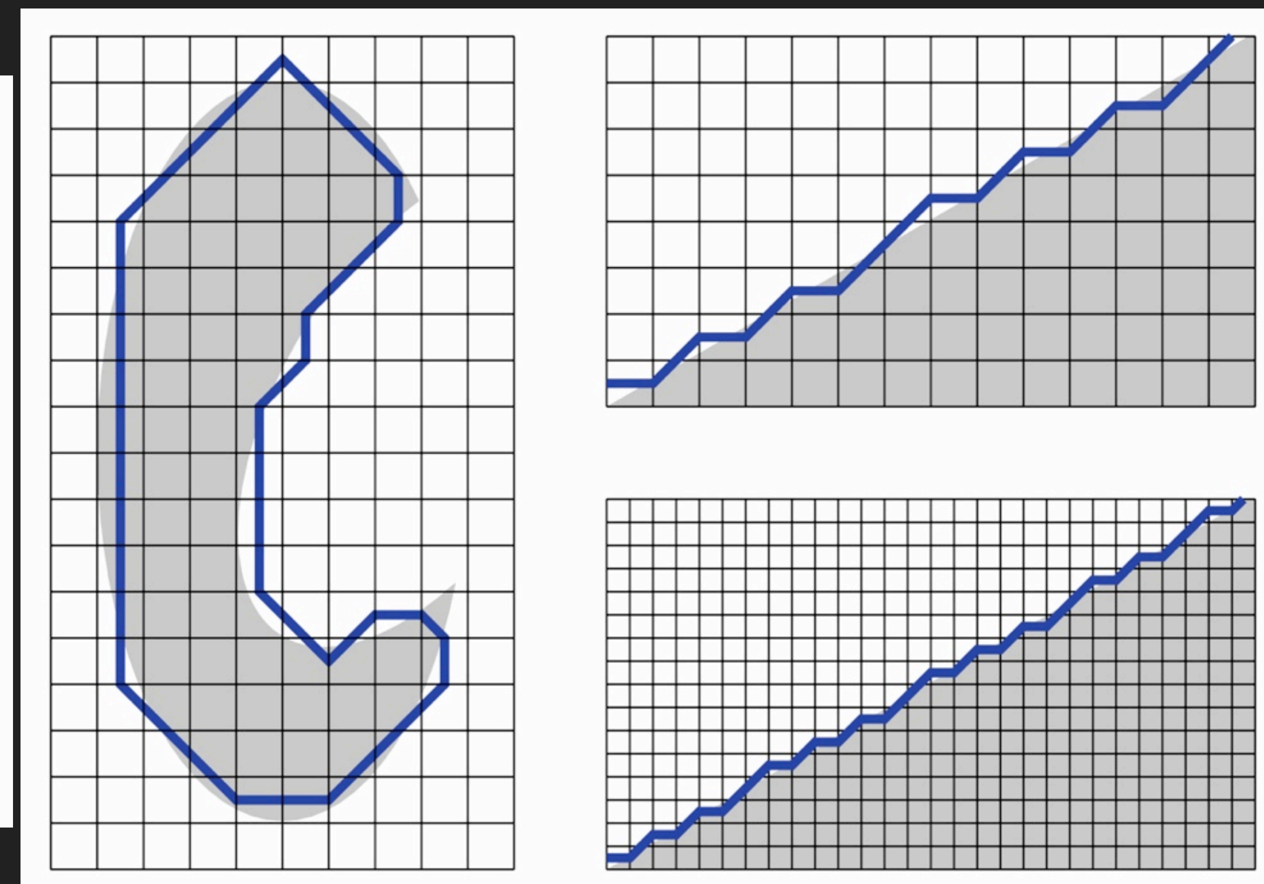


CONVERGENT



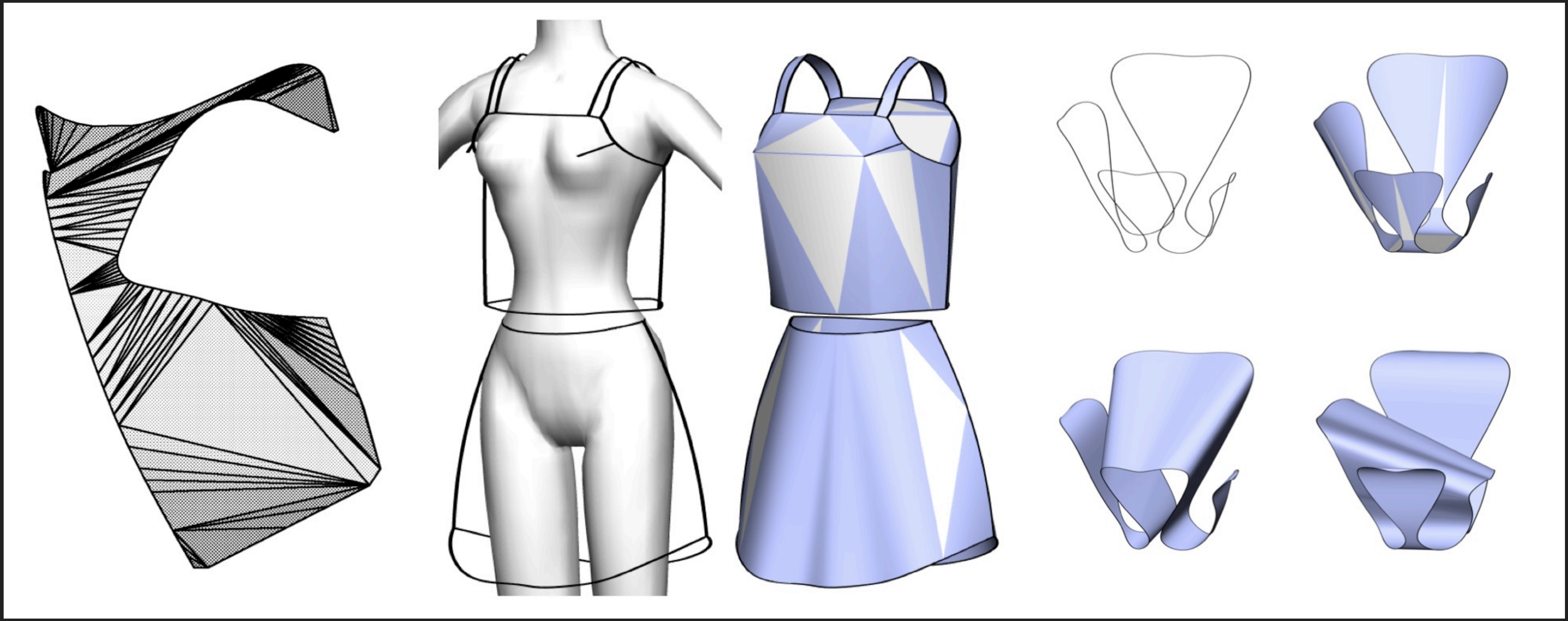
NONCONVERGENT

ONGOING: CONVERGENT BOUNDARY ESTIMATION

(LEFT) OPTIMIZED BOUNDARY CONVERGES AS INTERSAMPLE SPACING DECREASES
(RIGHT) LOCAL WINDOW BOUNDARY RECONSTRUCTION IS NONCONVERGENT

Prior Art: Relative Convex Hull

- SLOBODA, F. AND ZATKO, B. 2001. *On approximation of Jordan surfaces in 3D.* In Lecture Notes in Computer Science Volume 2243: Digital and Image Geometry, G. BERTRAND, A. IMIYA, AND R. KLETTE, Eds. Springer-Verlag, New York, NY, 365-386.
- Relative convex hull constructs a convergent boundary of a rasterized shape.



FREY, ROSE

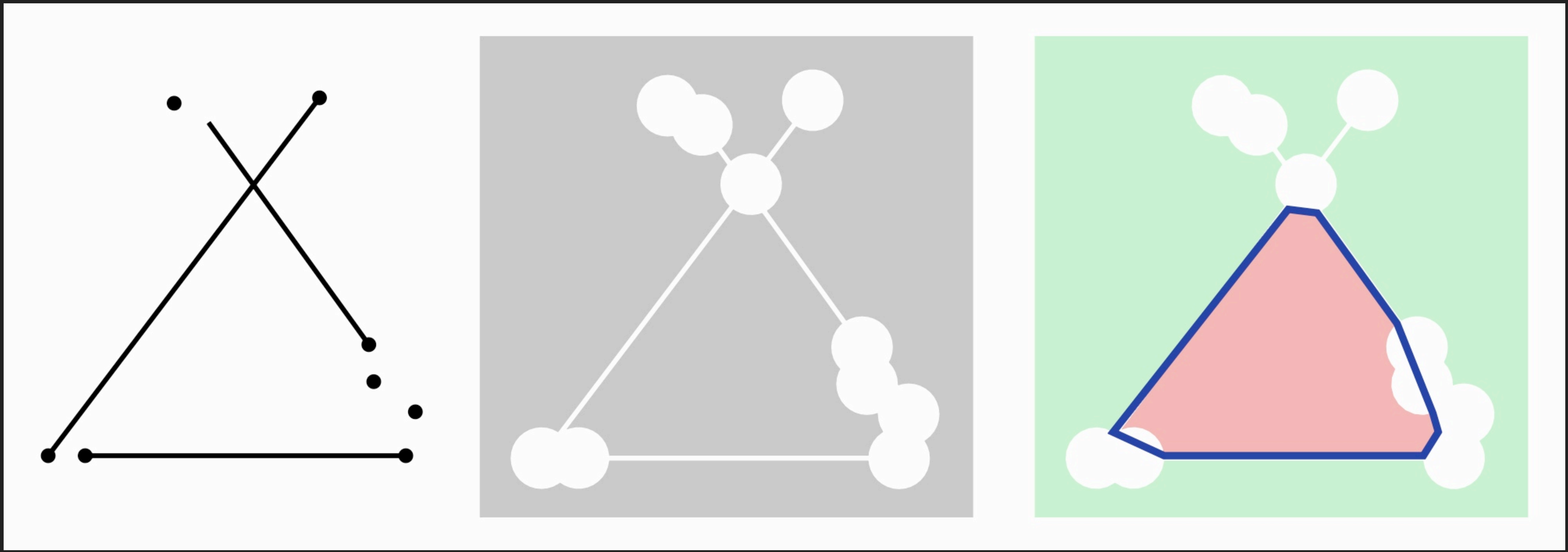
DEVELOPABLE SURFACES (ZERO GAUSSIAN CURVATURE)

PRIOR ART: MANUFACTURE

DEVELOPABLE TRIANGULATIONS OFFER AFFORDABLE MANUFACTURE

Prior Art: Developable

- FREY, W. 2002. *Boundary triangulations approximating developable surfaces that interpolate a space curve*. International Journal of Foundations of Computer Science 13, 285-302.
- ROSE, K., SHEFFER, A., WITHER, J. CANI, M.-P., AND THIBERT, B. 2007. *Developable surfaces from arbitrary sketched boundaries.* Proceedings of the 5th Eurographics Symposium on Geometry Processing, Barcelona, Spain, July 2007, A. BELYAEV AND M. GARLAND, Eds. Eurographics Association, Aire-la-Ville, Switzerland, 163-172.
- Developable triangulation of boundary curves.



CONJECTURE: POLYGONAL REPAIR

BOUNDARY SEPARATES COMPONENTS OF THE UNION OF EDGES AND DILATED VERTICES.

Prior Art: Polygonal Repair

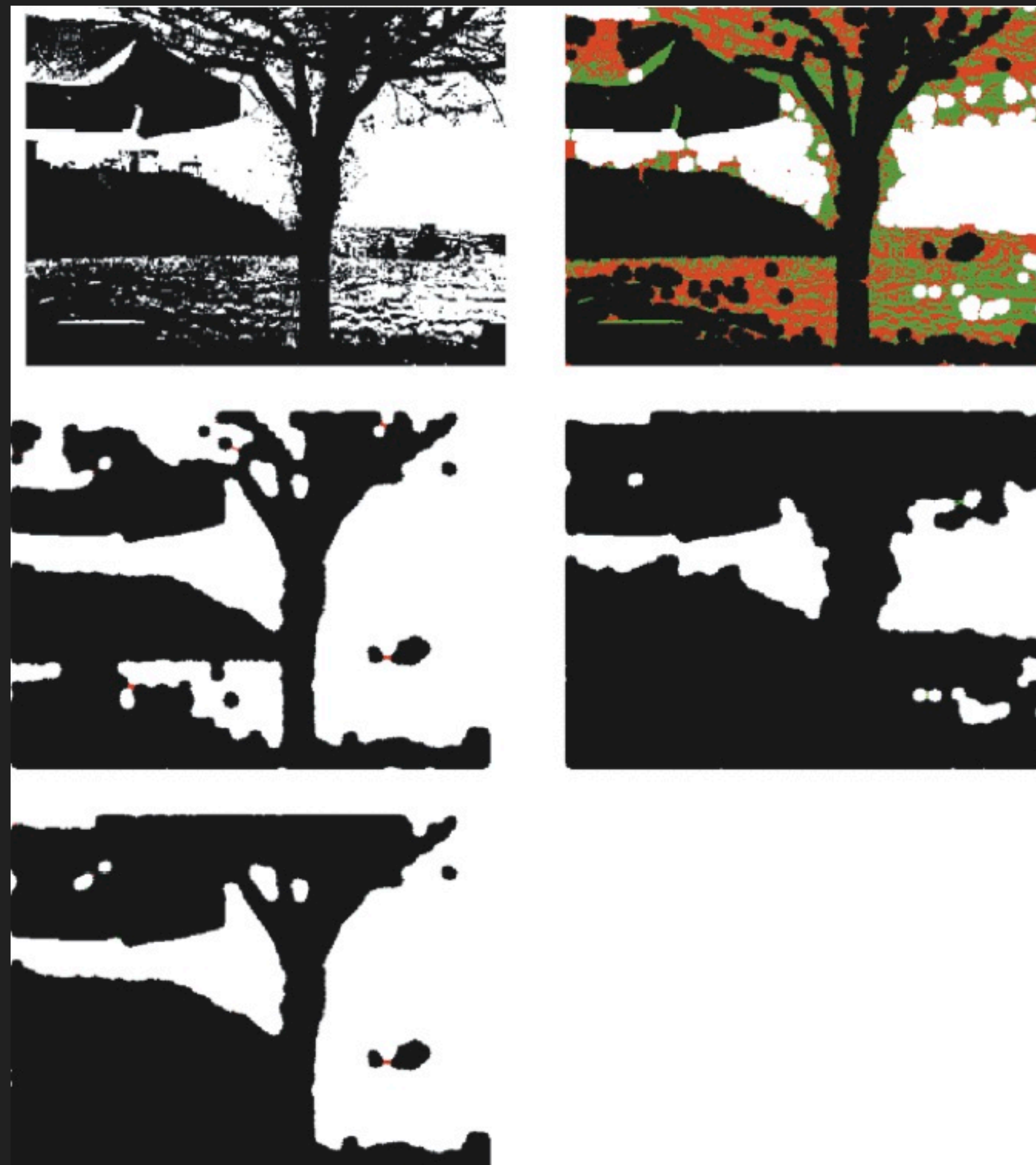
- MURALI, T. AND FUNKHOUSER, T. 1997. *Consistent solid and boundary representations from arbitrary polygonal data.* Proceedings of the 1997 Symposium on Interactive 3D Graphics.
- Constructing error-free solid and boundary representations from polygonal input.

Remaining Problems

- Asymmetric operators add or remove material.
- Inability to eliminate cusps and sharp features, with no curvature bound.
- No boundary simplification minimizing bumps and ripples.
- No topology management for organizing components.

Contributions

- Symmetric handling of interior and exterior sets, facilitating input fidelity.
- Guaranteed curvature bounds.
- Boundary simplified by minimizing measure.
- Control topology and specify canonical topology.

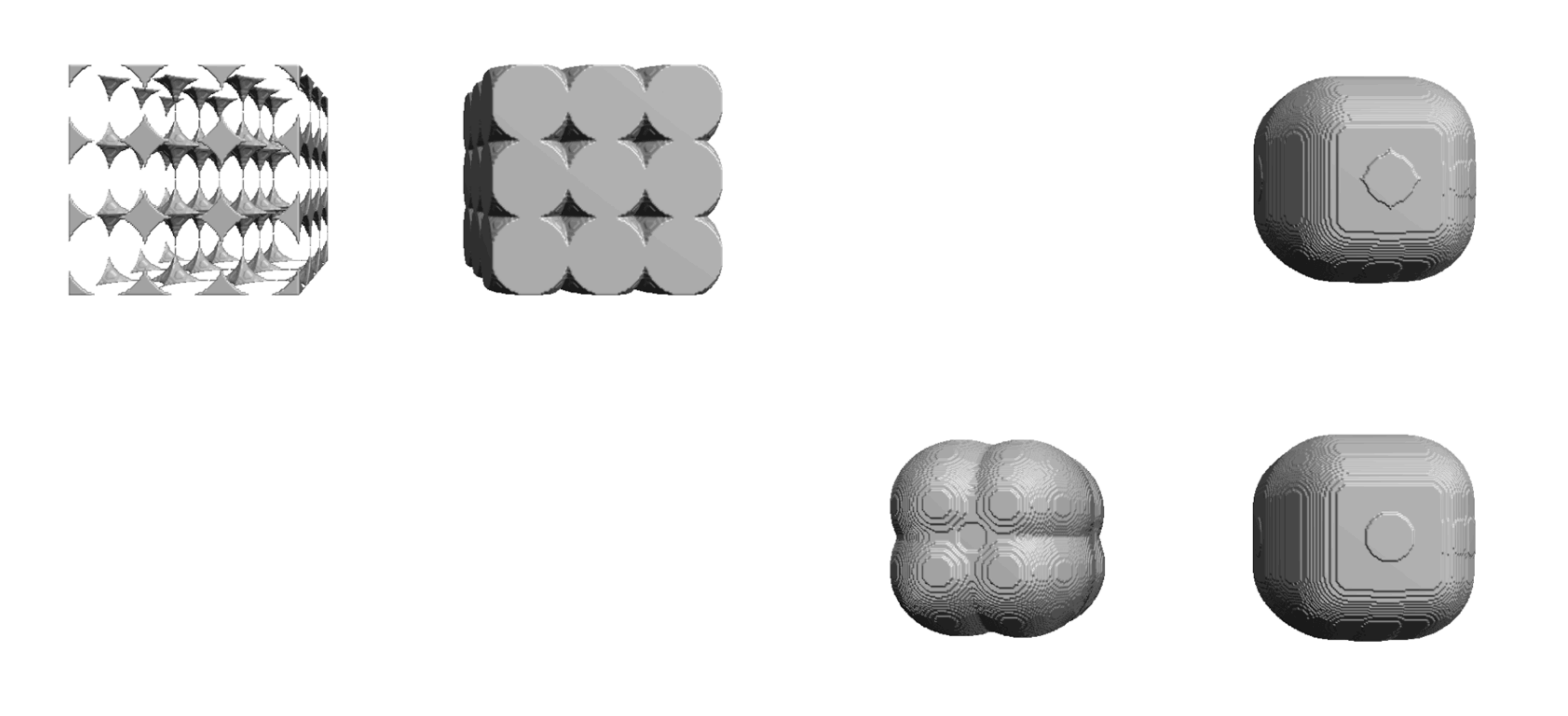


PUBLISHED: MASON

SYMMETRIC REGULARIZATION THROUGH MORPHOLOGICAL OPENING AND CLOSING

INPUT

CLOSE/OPEN



OPEN/CLOSE

MASON

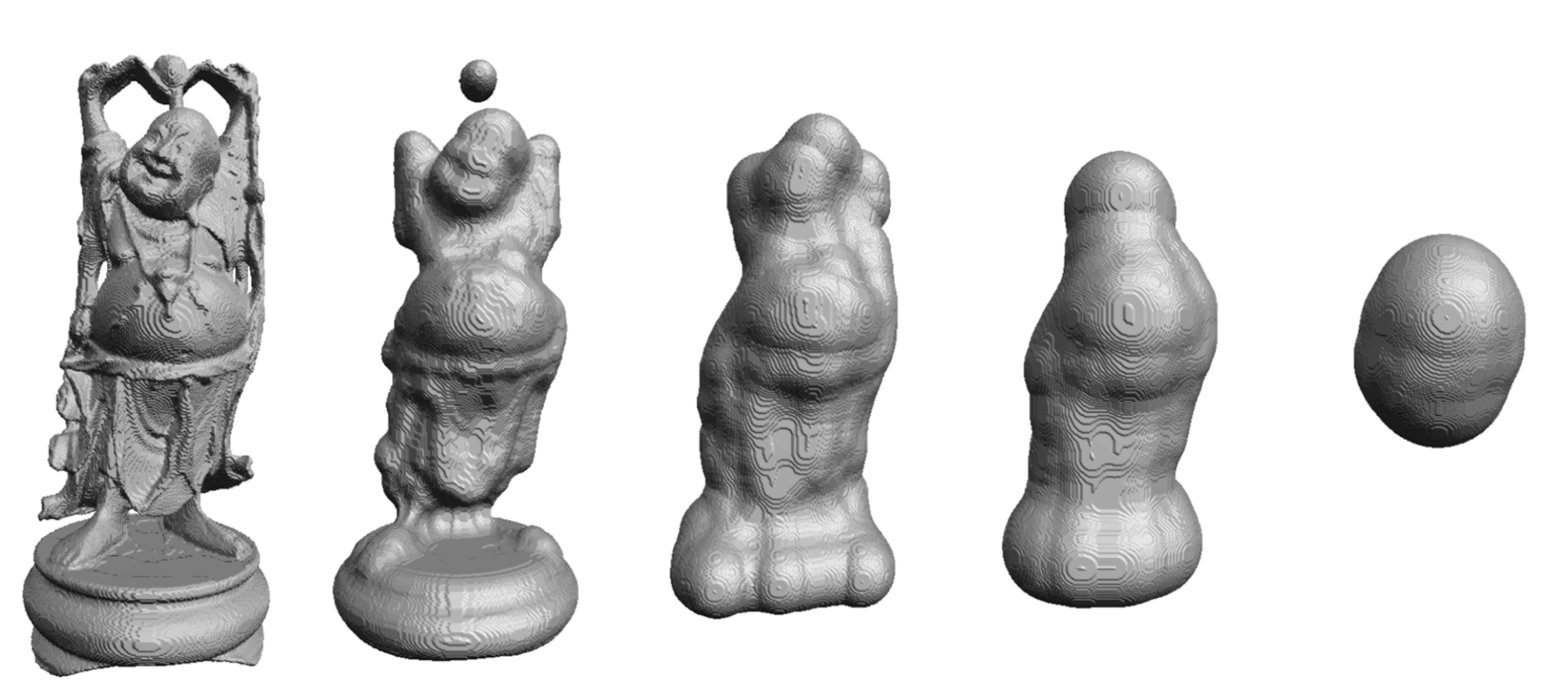
PUBLISHED: MASON

SYMMETRIC REGULARIZATION THROUGH MORPHOLOGICAL OPENING AND CLOSING

Published:Mason

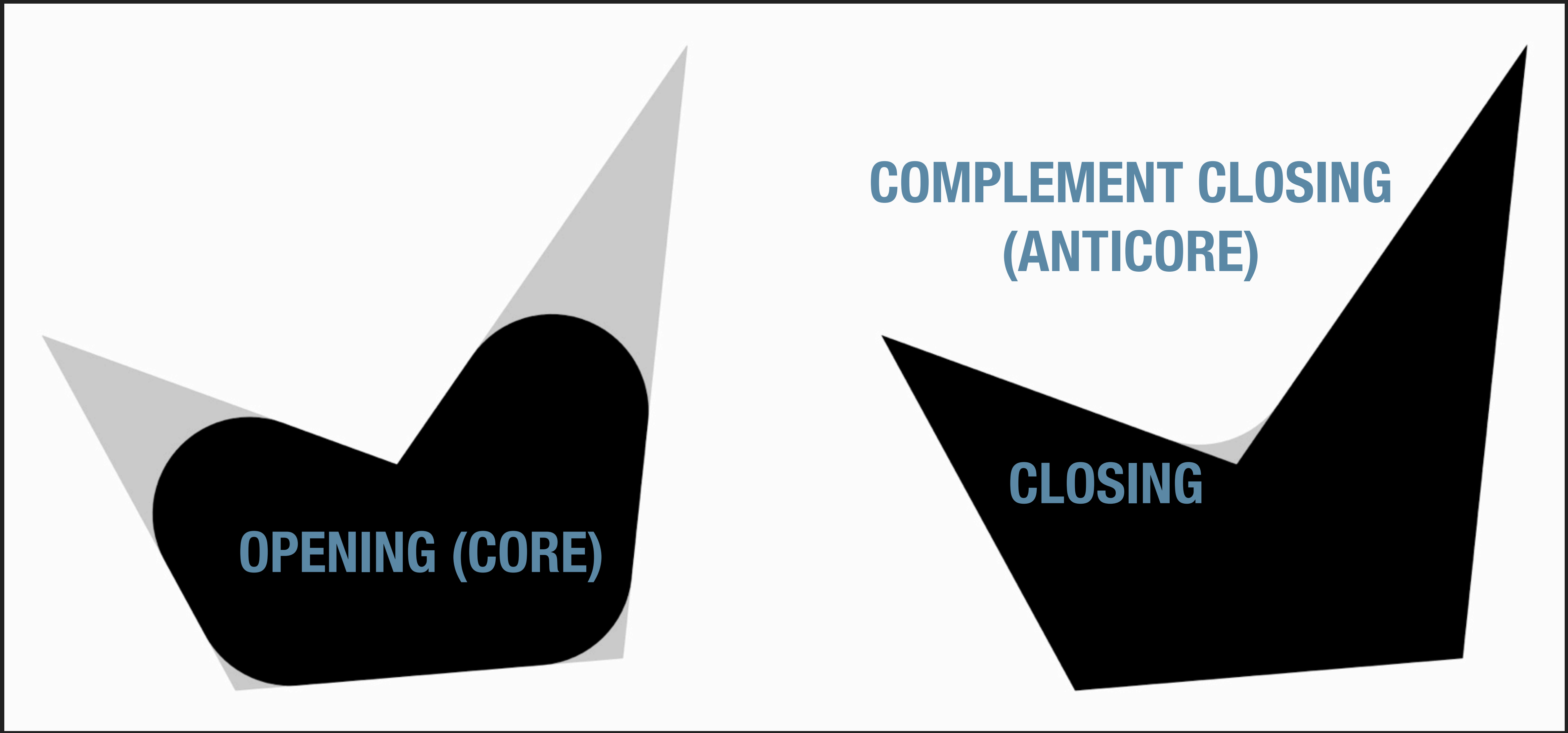
- WILLIAMS, J. AND ROSSIGNAC, J. 2005. *Mason: Morphological simplification.* Graphical Models 67, 285-303.

MASON RADIUS INCREASING LEFT TO RIGHT



Review: Mathematical Morphology

- Use a radius to define a set of balls in relation to an input set.
- Construct output sets using union, intersection, and complement.
- In the limit, morphological operators resemble interior, closure, and boundary.



PRIOR ART: OPENING AND CLOSING WITH A BALL (CORE AND ANTICORE)

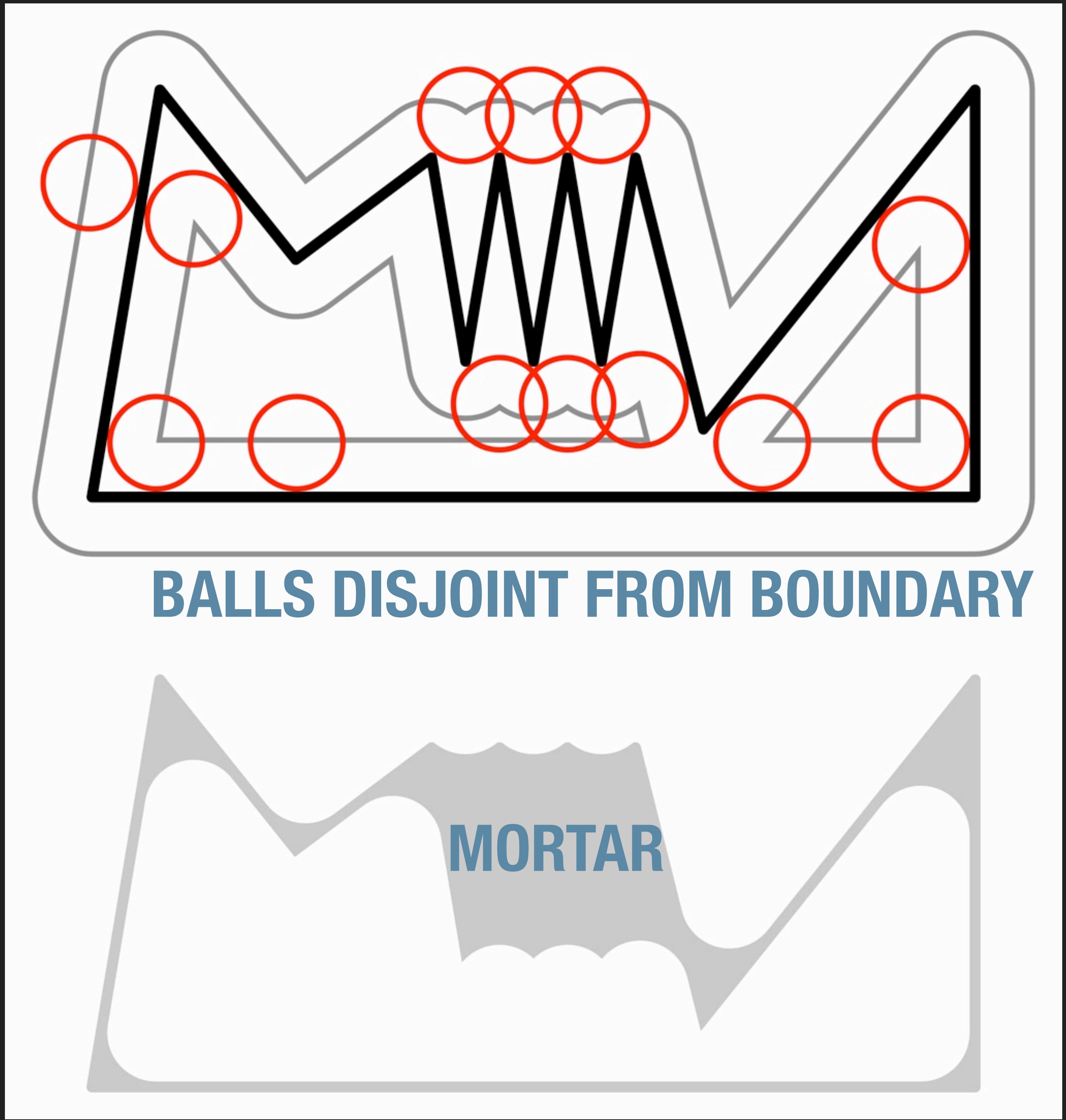
(LEFT) OPENING REMOVES GRAY, LEAVING BLACK (OPENING IS THE CORE)
(RIGHT) CLOSING ADDS GRAY TO BLACK (COMPLEMENT OF CLOSING IS THE ANTICORE)

Prior Art: Mathematical Morphology

- SERRA, J. 1982. *Image Analysis and Mathematical Morphology, Volume 1.* Academic Press, New York, NY.
- ROSSIGNAC, J. 1985. *Blending and Offsetting Solid Models.* Doctoral dissertation, University of Rochester, Rochester, NY.
- Morphology applied to mining and modeling.

Published: Mortar

(Above) Boundary of the mortar defined by red circles rolling along the black polygon boundary. (Below) Mortar shown in gray, separating the core and anticore.



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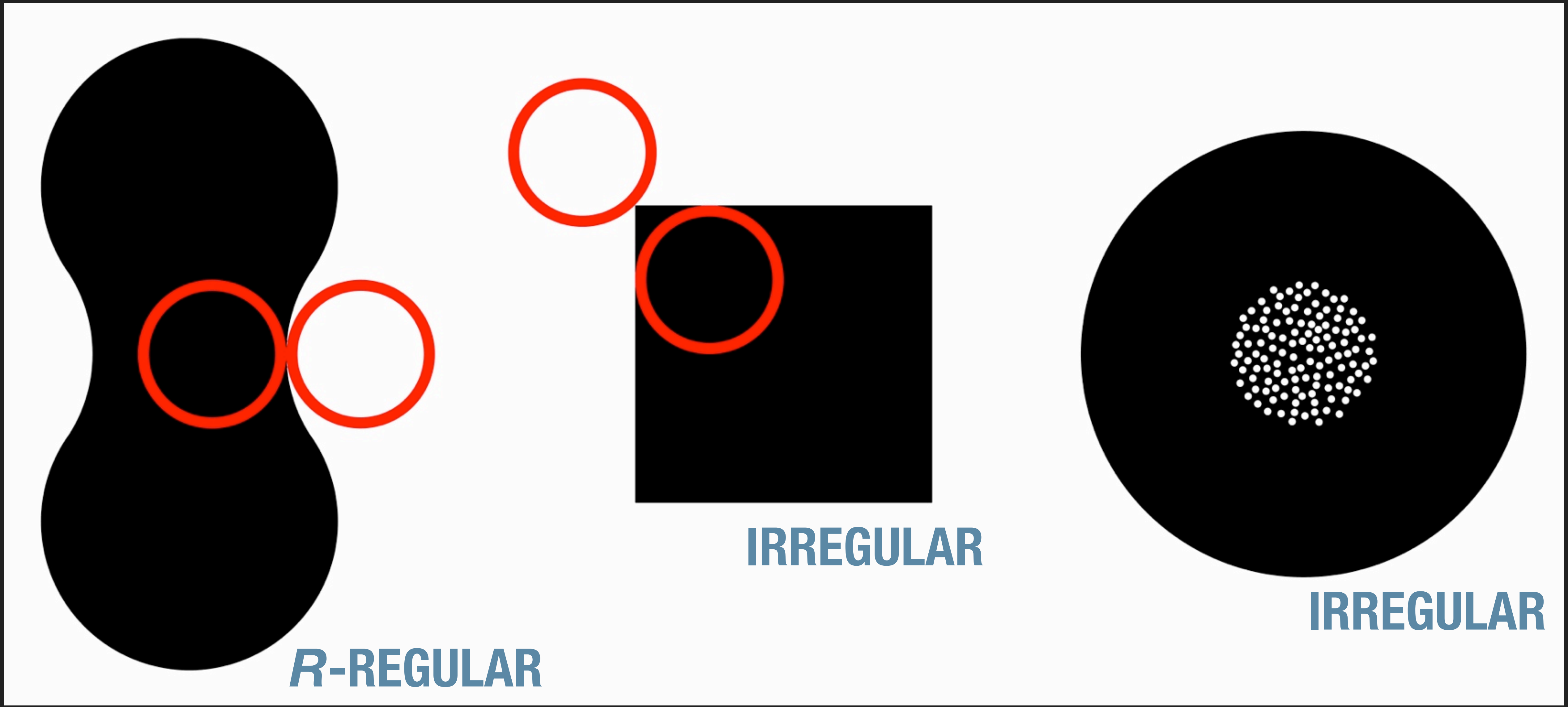


PRIOR ART: *R*-REGULAR SET

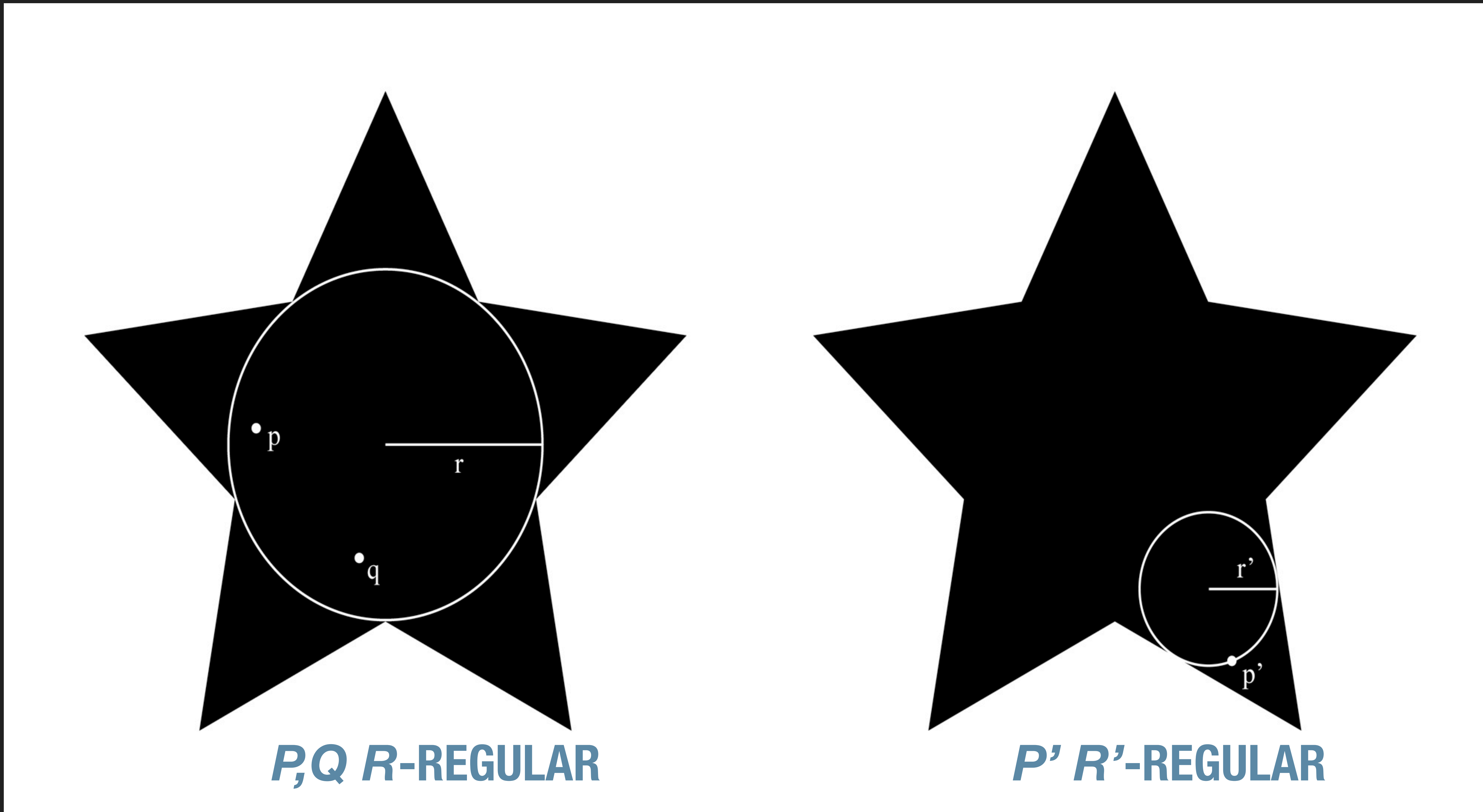
A CLOSED SET *S* WITH MANIFOLD BOUNDARY IS *R*-REGULAR IF AND ONLY IF...
(LEFT) *R*-REGULAR (MIDDLE) *R*-IRREGULAR (RIGHT) *R*-IRREGULAR

Prior Art: r-Regular Sets

- ATTALI, D. 1997. *r-Regular shape reconstruction from unorganized points.* Proceedings of the 13th Annual Symposium on Computational Geometry, Nice, France, June 1997, J.-D. BOISSONNAT, Ed. ACM, New York, NY, 248-253.

- SERRA, J. 1982. *Image Analysis and Mathematical Morphology, Volume 1.* Academic Press, New York, NY.

- Definition and properties of regularity, with relation to samples.



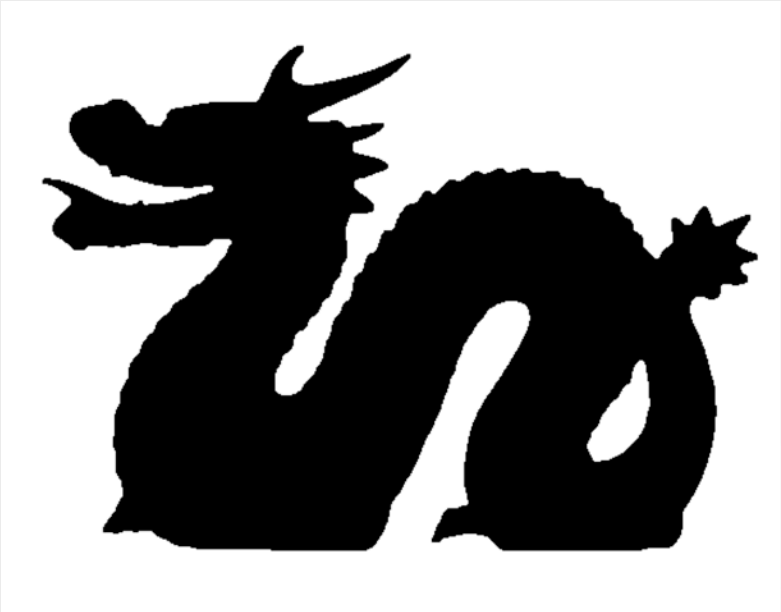
PUBLISHED: REGULARITY TRANSFORM

REGULARITY IS THE LARGEST BALL IN A SET CONTAINING A POINT
(LEFT) POINTS *P* AND *Q* ARE *R*-REGULAR (RIGHT) POINT *P'* IS *R'*-REGULAR

Distance Transform vs. Regularity Transform

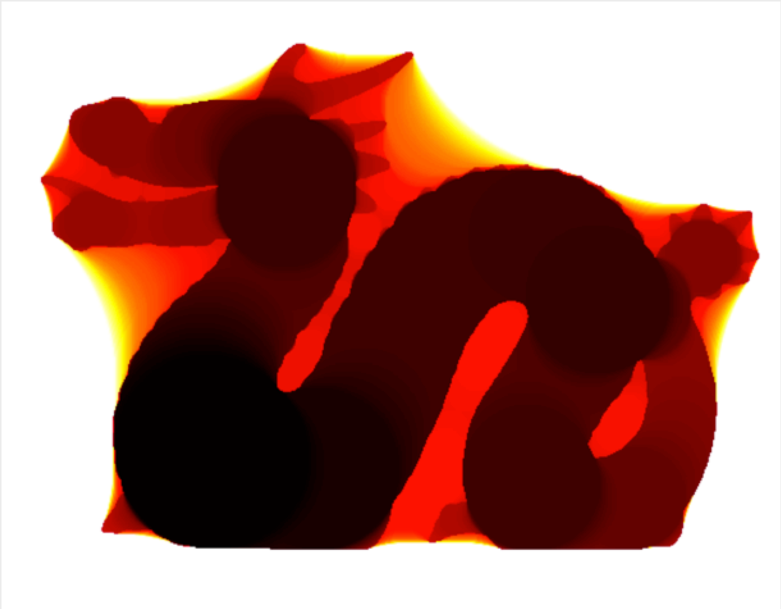
- Distance transform measures distance to a set.
- Regularity measures sizes of balls disjoint from a set.
- Maximal disks centered on distance singularities.

INPUT



DISTANCE TRANSFORM

REGULARITY TRANSFORM



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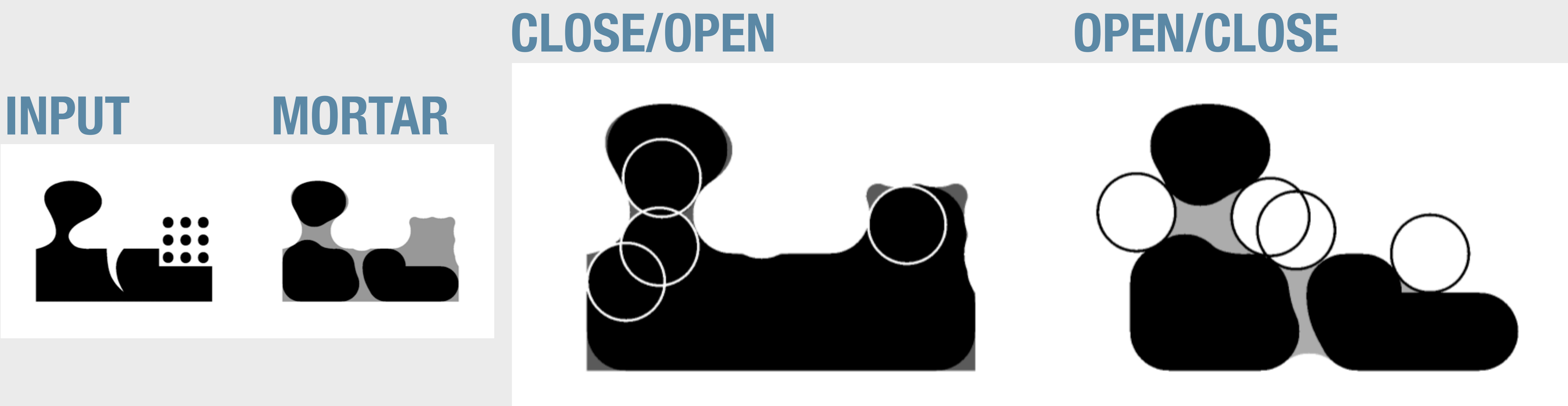
Motivating Mason

- Opening/closing and closing/opening nearly regularize by modifying the mortar.
- First opening reduces area in 2D or volume in 3D, while first closing adds.
- Changing output measure increases difference between input and output.



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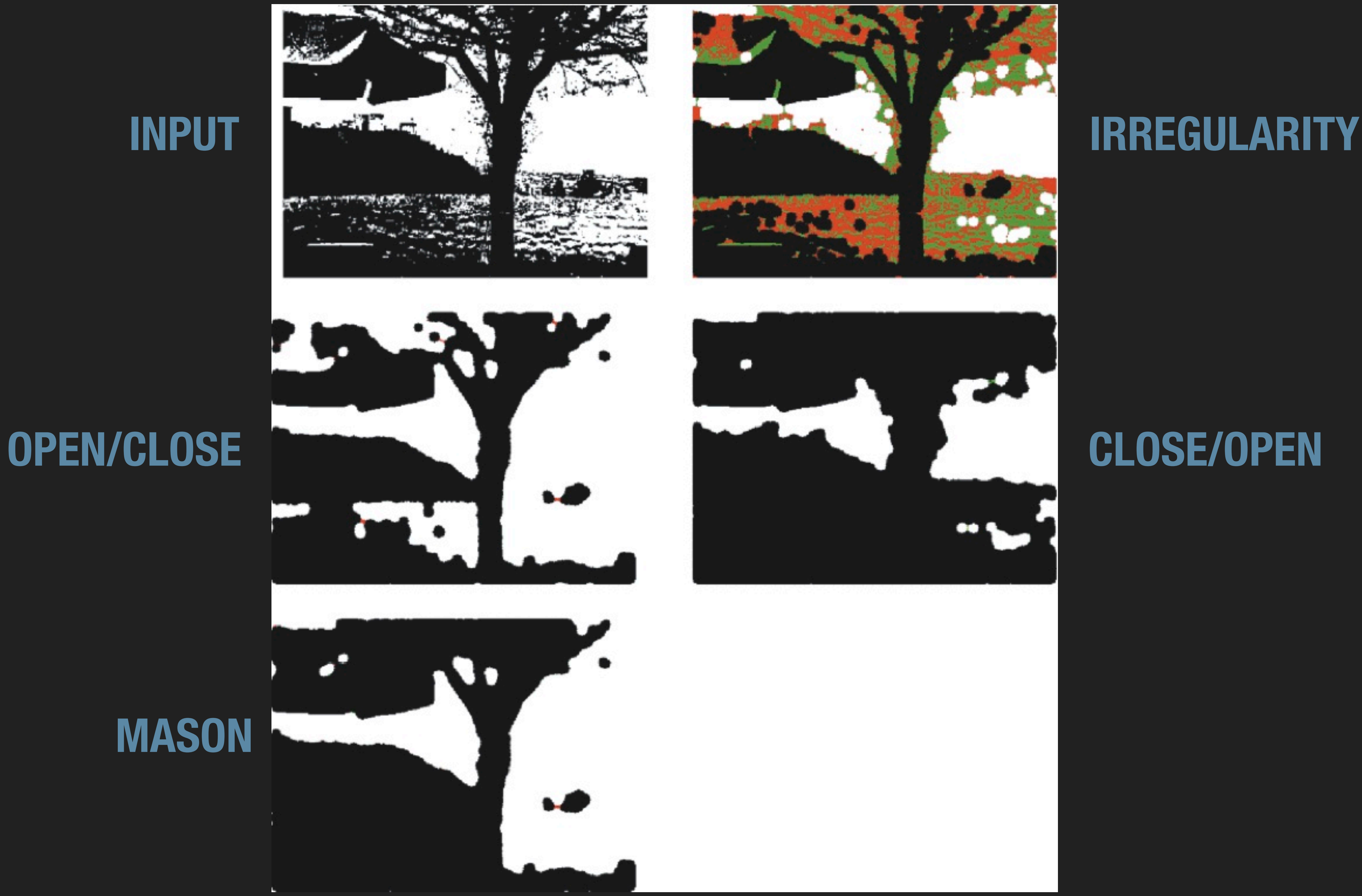
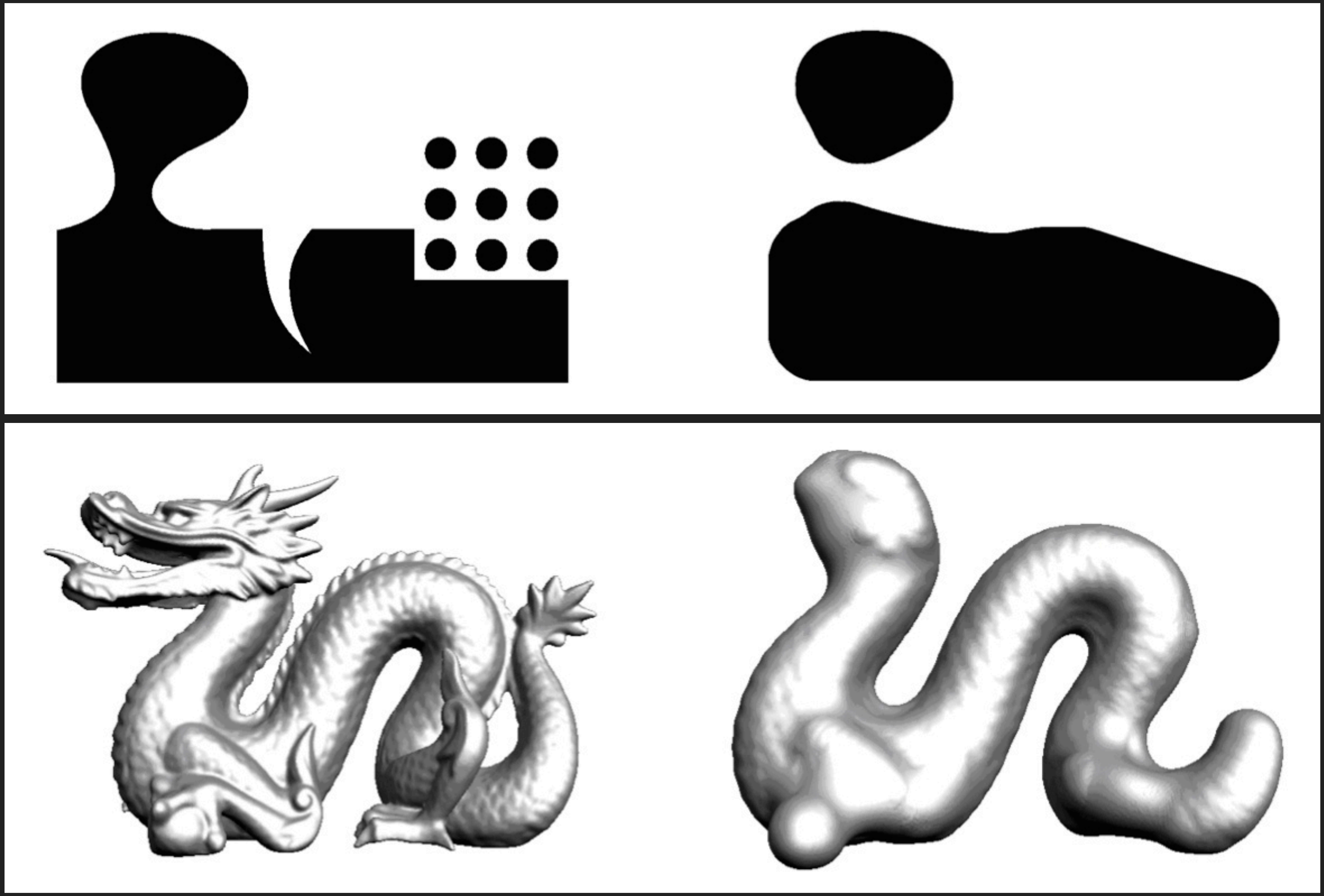
Mason Definition and Desirable Properties

- For each mortar component, mason chooses open/close or close/open to minimize symmetric difference with the input.
- Mason reduces total symmetric difference.
- Mason is symmetric and nearly regular.



MINIMIZING SYMMETRIC DIFFERENCE

MASON YIELDS SMALLER SYMMETRIC DIFFERENCE THAN OPEN/CLOSE OR CLOSE/OPEN.



PUBLISHED: TIGHTENING

MINIMIZING BOUNDARY MEASURE AND MEAN CURVATURE THROUGH MEAN CURVATURE FLOW

Published: Tightening

- WILLIAMS, J. AND ROSSIGNAC, J. 2004. *Tightening: Curvature-limiting morphological simplification.* Fall Workshop on Computational Geometry.

- WILLIAMS, J. AND ROSSIGNAC, J. 2005. *Tightening: Curvature-limiting morphological simplification.* ACM Symposium on Solid and Physical Modeling.

- WILLIAMS, J. AND ROSSIGNAC, J. 2007. *Tightening: Morphological simplification.* International Journal of Computational Geometry and Applications 17, 487-5

Motivating Tightening

- Mason is symmetric and nearly regular but can have cusps.
- Regular sets have principal curvatures bounded by -1/*r* to 1/*r*, but regularization might not confine changes to the mortar.
- Seek a symmetric curvature bound in the mortar.

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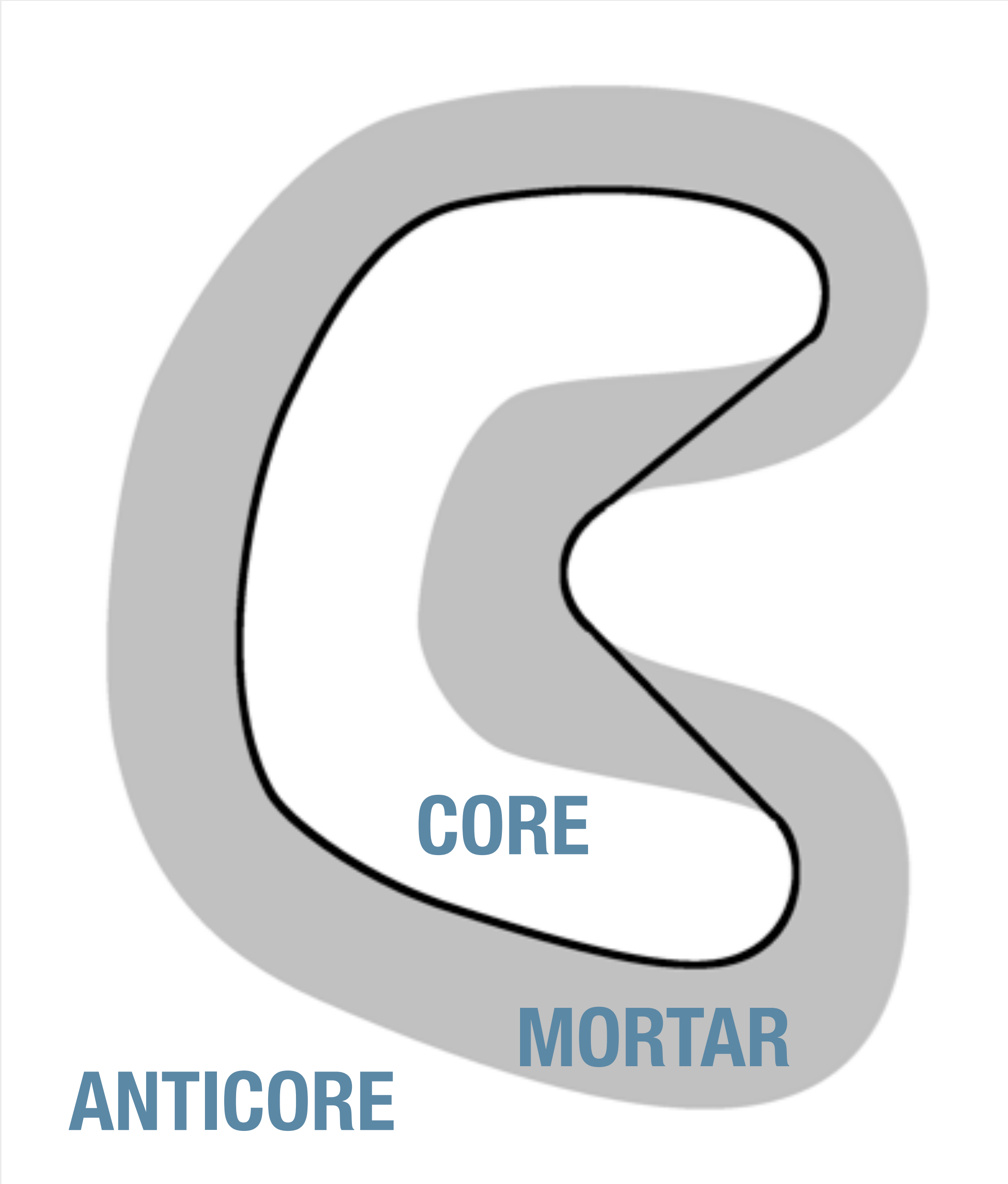
Tightening Definition

- Candidates contain the core and exclude the anticore.
- Tightening minimizes boundary measure out of candidates that remain to be candidates throughout constrained isotopy.



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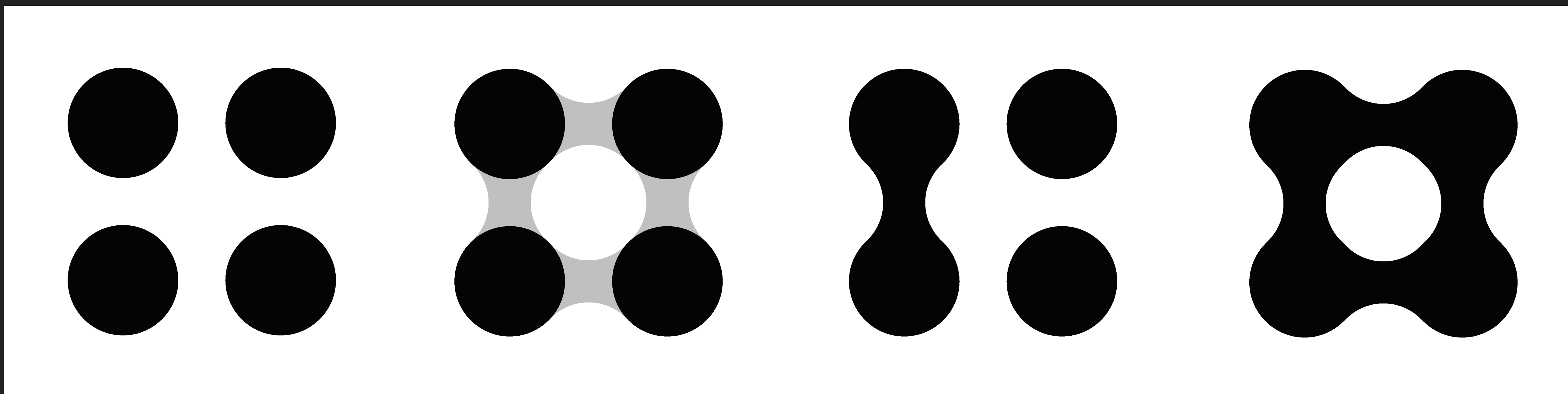

INPUT (TIGHTENING) | MORTAR (GRAY) | TIGHTENING | TIGHTENING

MULTIPLE *R*-TIGHTENING TOPOLOGIES

MULTIPLE TIGHTENING BOUNDARIES LOCALLY MINIMIZE BOUNDARY MEASURE.

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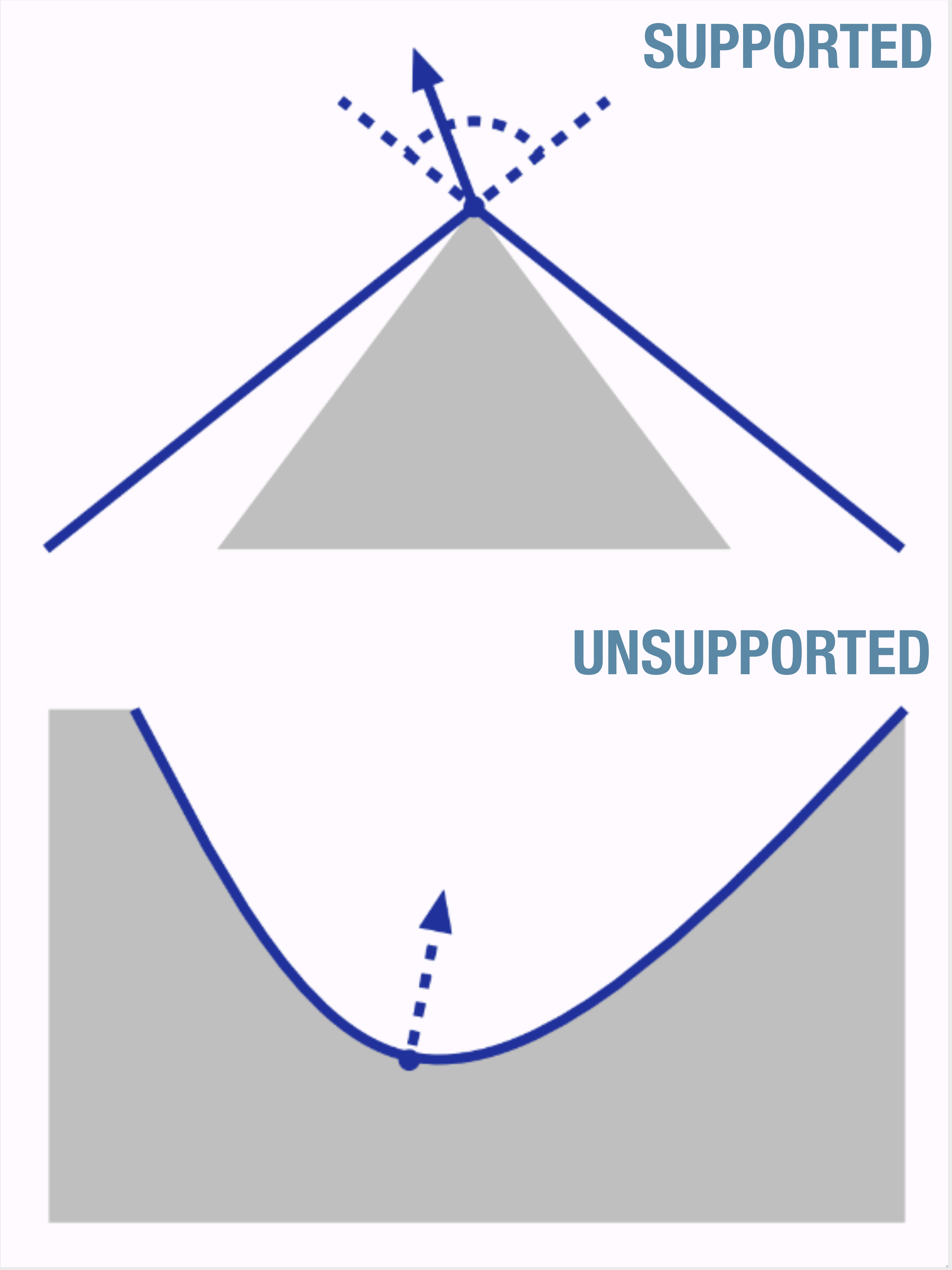
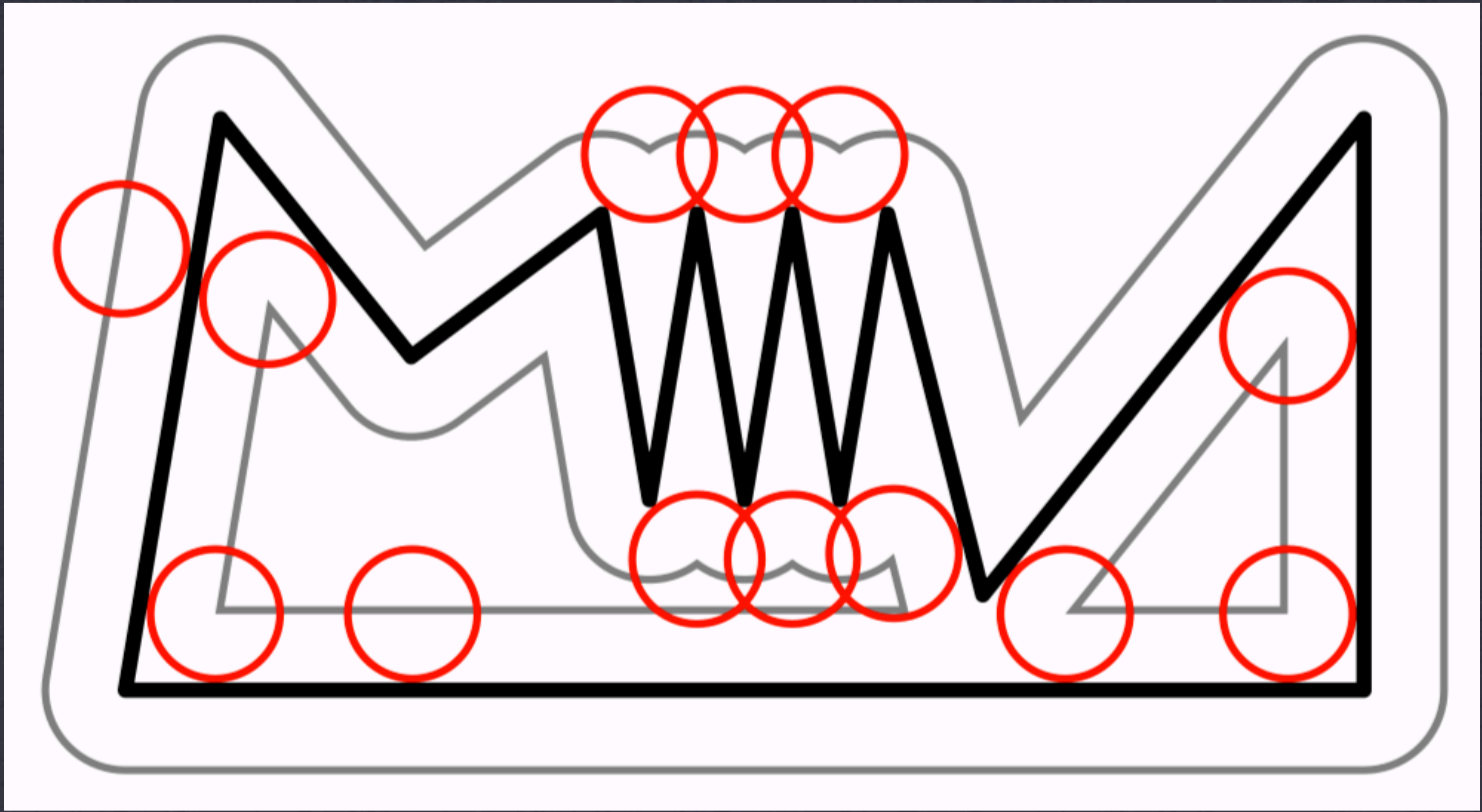
Support

- Convexity normal of *S* is scaled by locally convex two-dimensional sections.
- Support of a convexity normal of *S* is its maximum dot product over the set of convex normals to *T*.

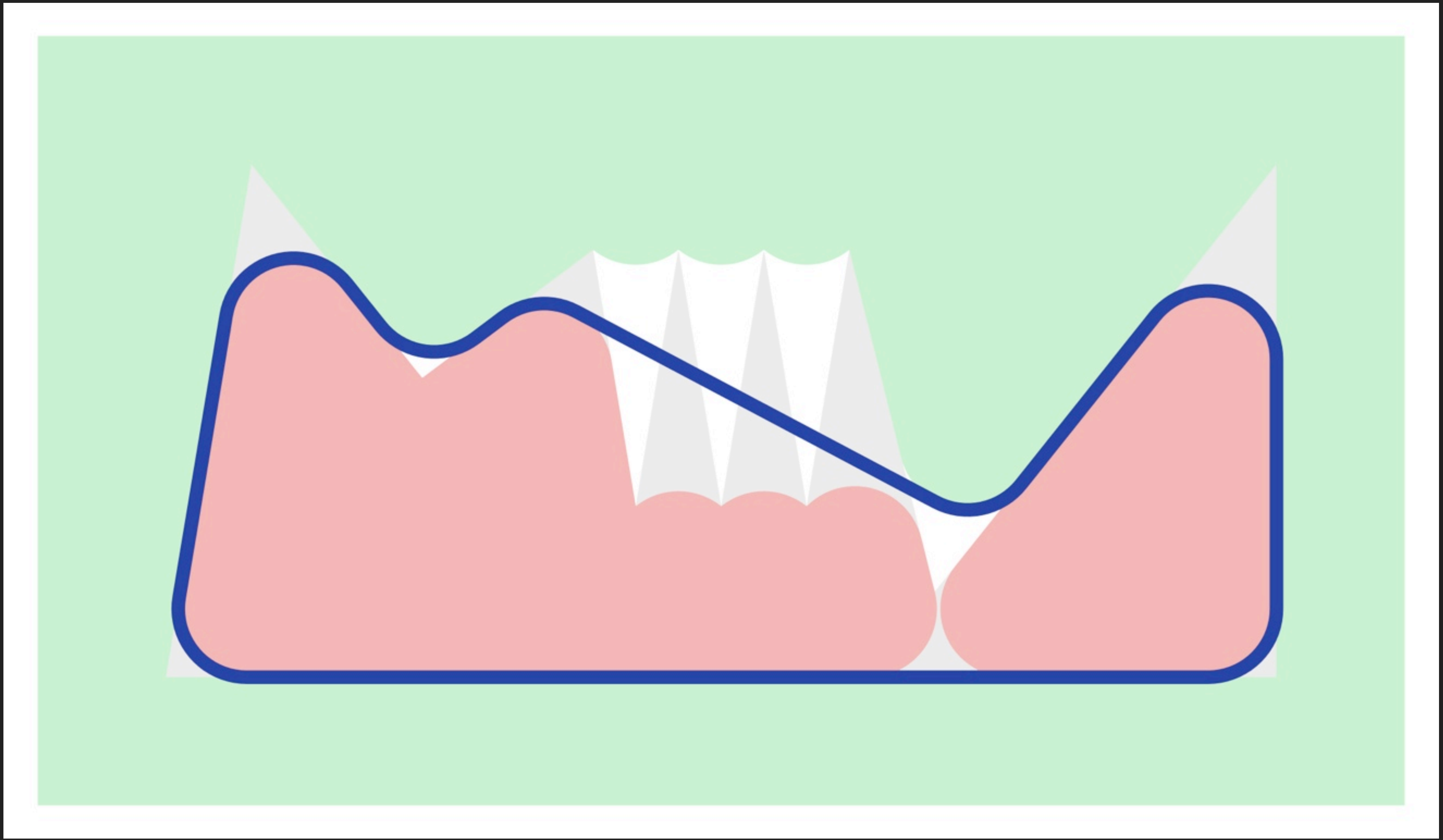


Tightening: Desirable Properties

- Tightening is symmetric.
- Local boundary area minimization.
- Unsupported boundary consists of minimal surface patches.
- Supported mean curvature has value [-1/*r*,1/*r*].
- Unsupported mean curvature is zero.



POLYGON BOUNDARY IS BLACK; BALLS ARE OUTLINED RED
BALLS INSIDE FORM CORE; BALLS OUTSIDE FORM ANTICORE



MEAN CURVATURE BOUND

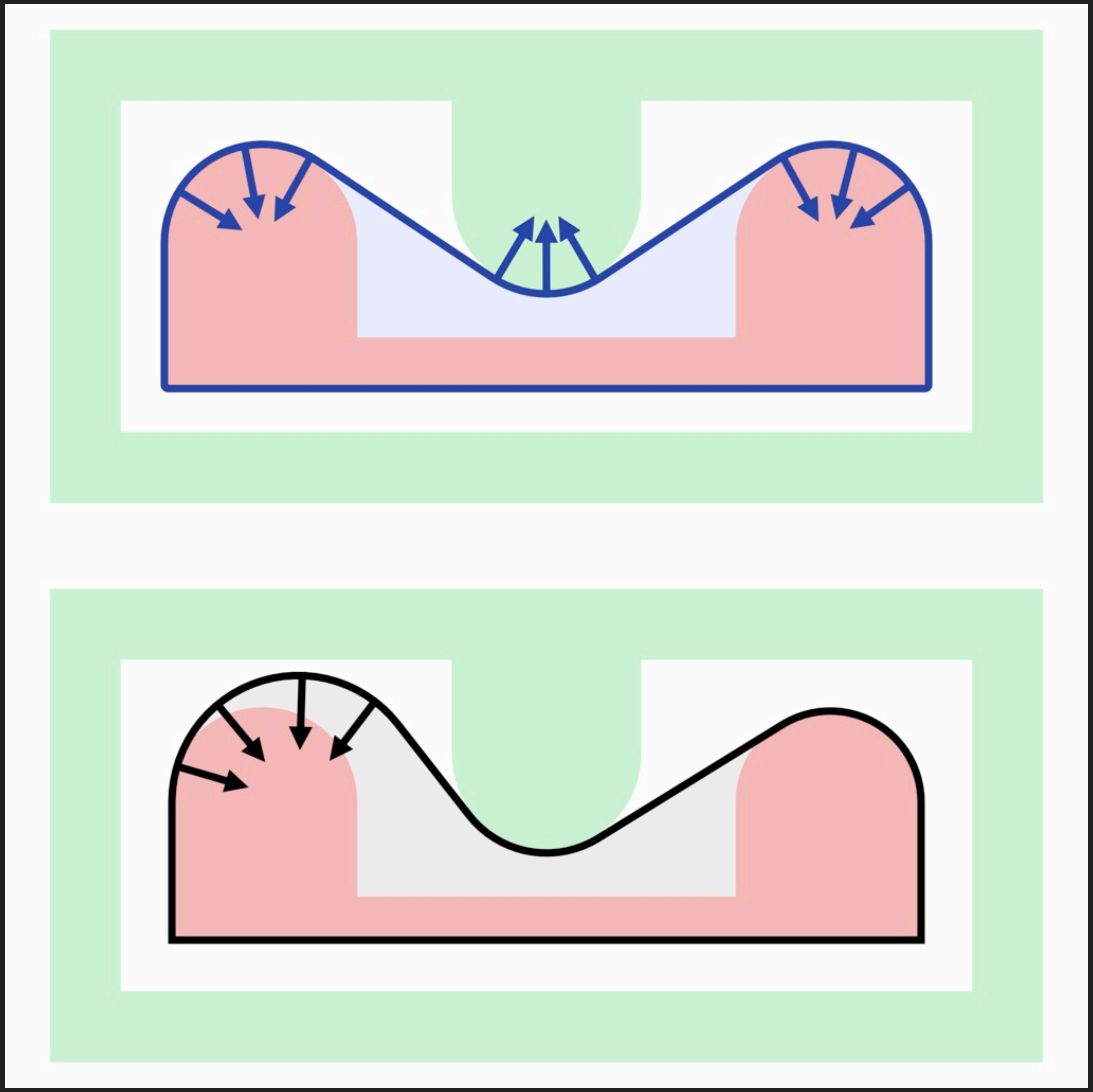
***R*-TIGHTENING MEAN CURVATURE HAS MAGNITUDE LESS THAN OR EQUAL TO 1/*R*.**

Constrained Mean Curvature Flow

- Given a candidate, we obtain a tightening by applying constrained mean curvature flow.
- A mean curvature normal at a boundary point is a normal scaled by the mean of principal curvatures at that point.
- We move the boundary along the field of mean curvature normals, which is along the boundary measure gradient.
- Motion stops at constraints or where the mean curvature is zero.

STABLE
(TIGHTENING)

UNSTABLE
(CANDIDATE)



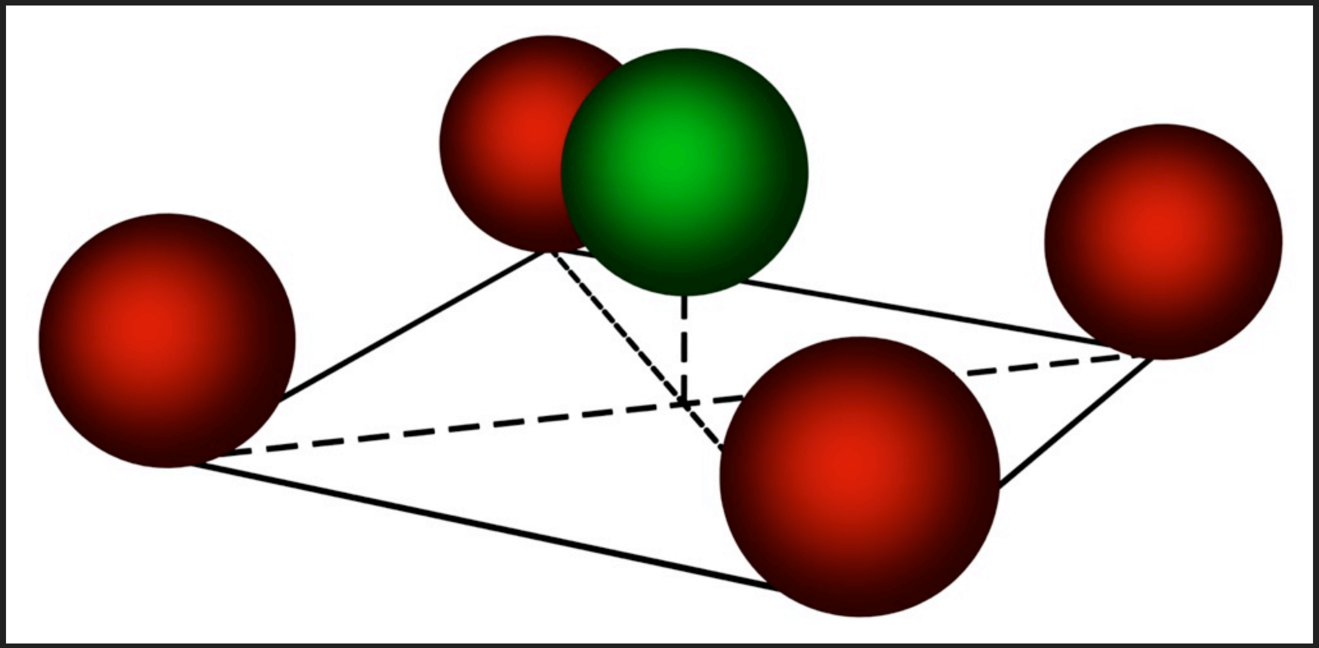
MEAN CURVATURE FLOW STABILITY

FLOW IS STABLE IF AND ONLY IF NORMALS POINT INWARD AT CONSTRAINT BOUNDARIES.

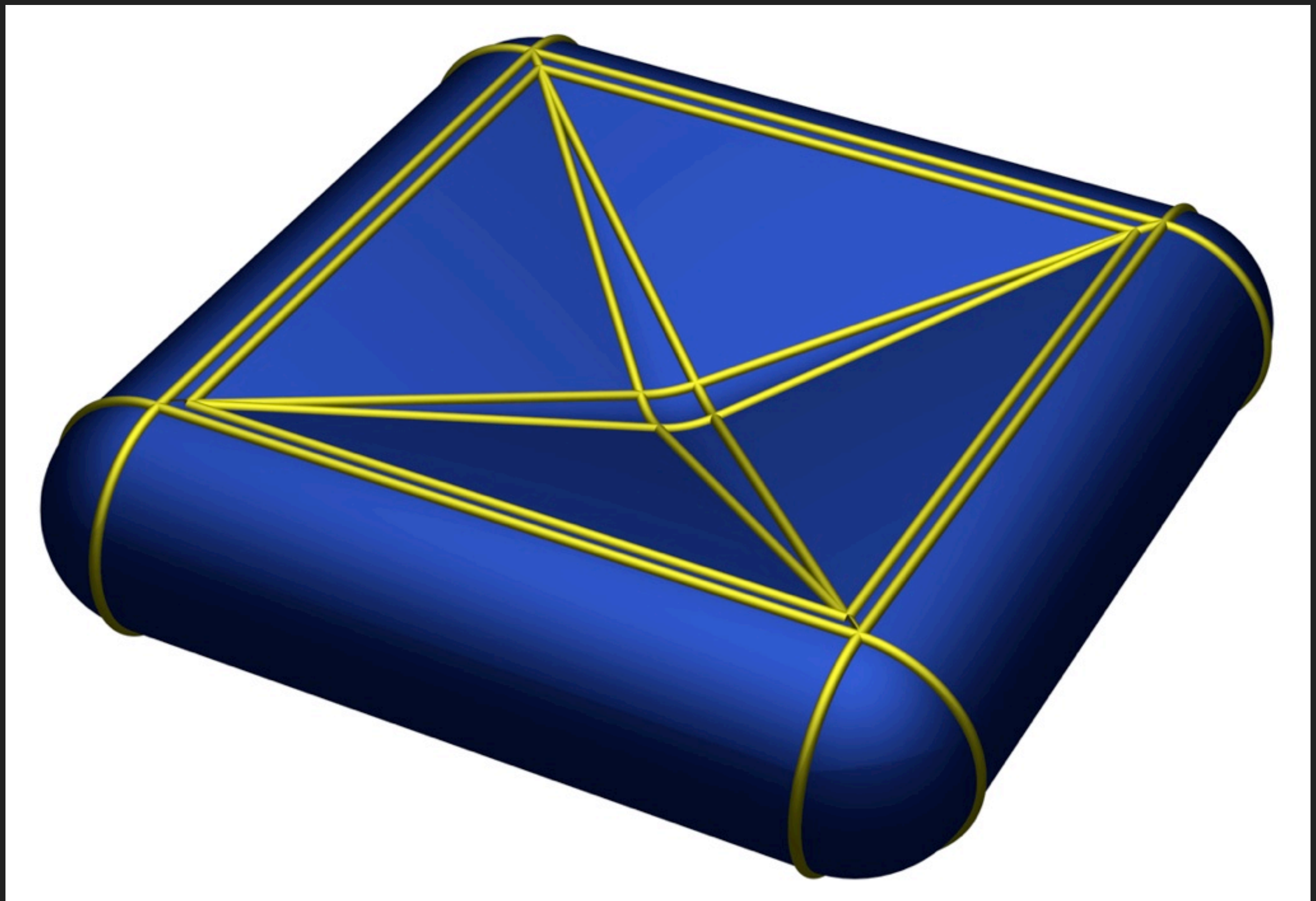
Prior Art: Mean Curvature

- CHOPP, D. 1993. *Computing minimal surfaces via level set curvature flow.* Journal of Computational Physics 106, 77-91.
- GRAYSON, M. 1987. *The heat equation shrinks embedded plane curves to round points.* Journal of Differential Geometry 26, 285-314.
- SETHIAN, J. 1999. *Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science*. Cambridge University Press, New York, NY.

- Computing mean curvature flow.



CONTAIN RED
EXCLUDE GREEN



BOUNDARY WITH
PATCH OUTLINES

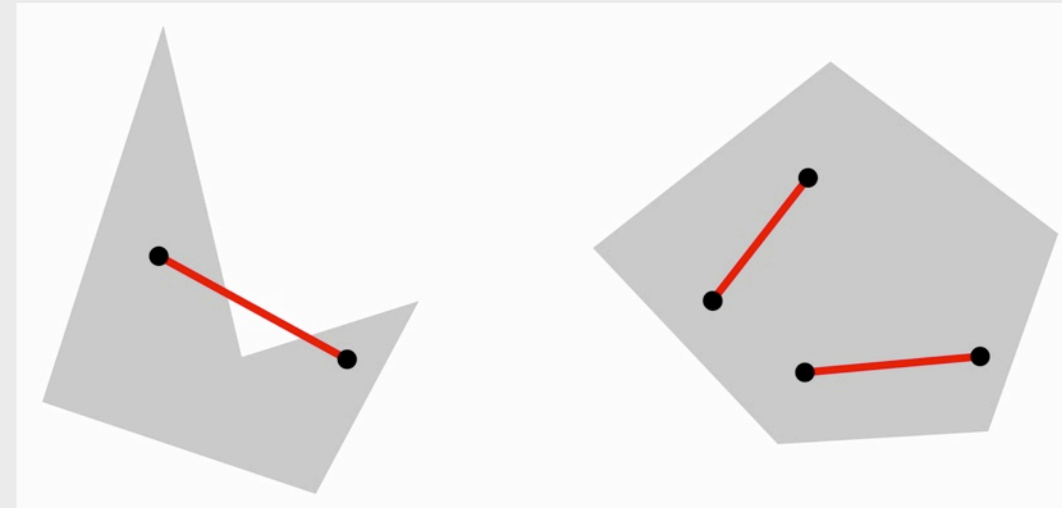
UNPUBLISHED: TIGHT HULL

SYMMETRIC TIGHT HULL GENERALIZATION MINIMIZING SLACK AND BOUNDARY

Convex Hull

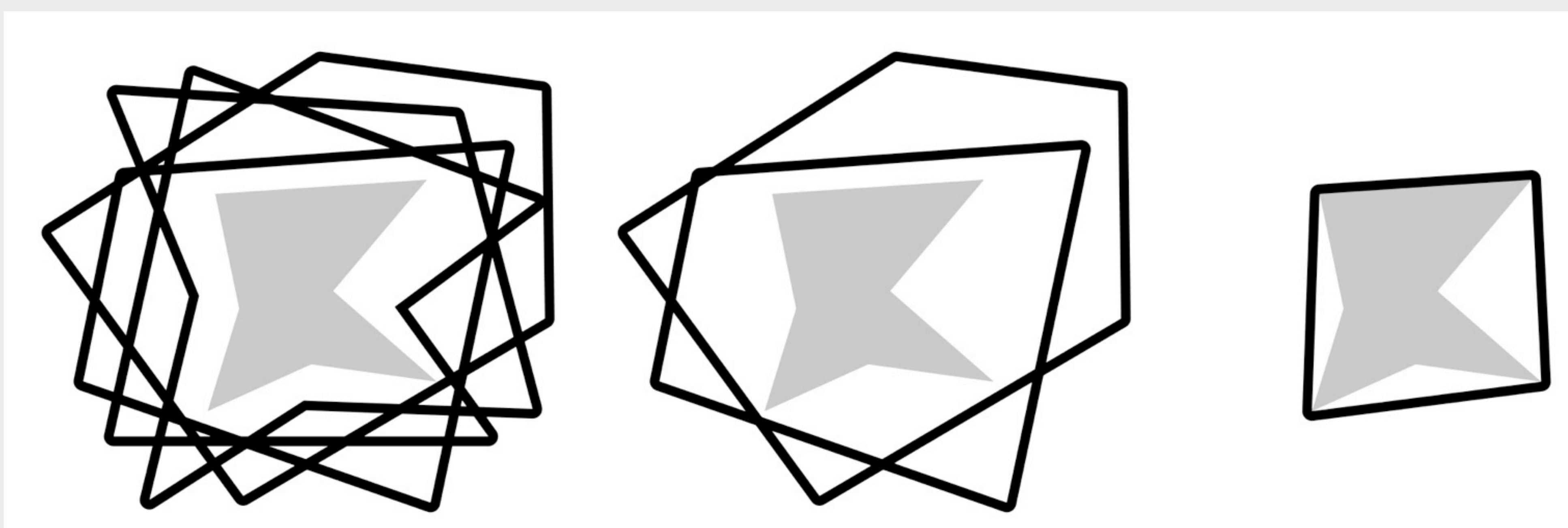
- A *convex set* contains every segment connecting some pair of points in the set.

- The *convex hull* of set *S* is the intersection of all convex sets containing *S*.



NOT CONVEX CONVEX

CONTAINING INPUT CONVEX CONTAINING CONVEX HULL



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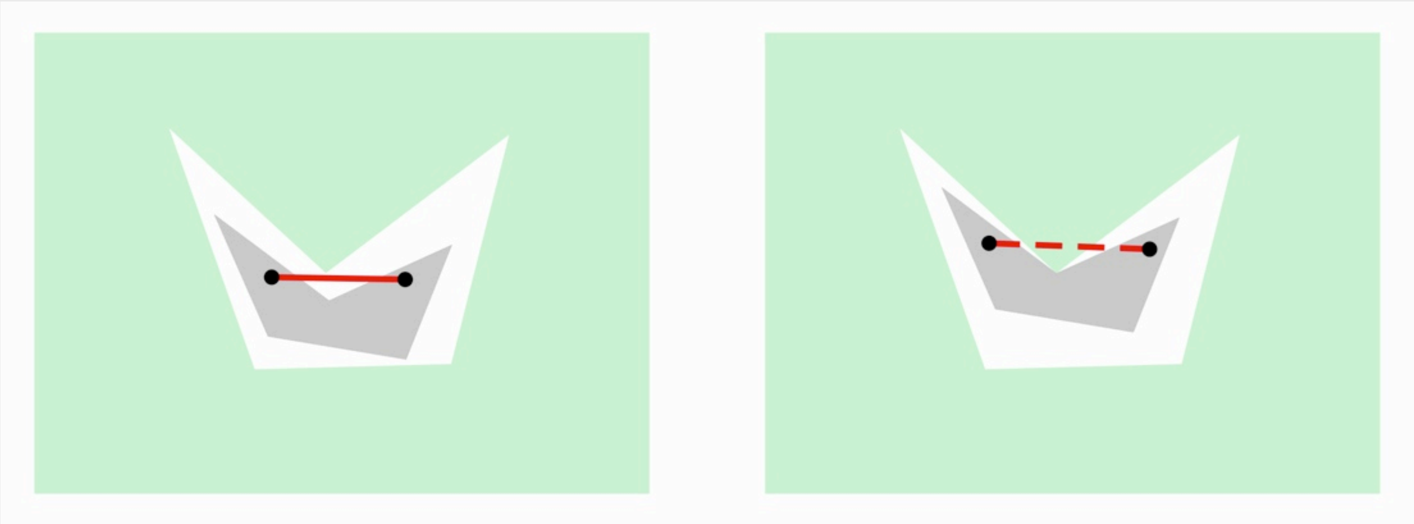
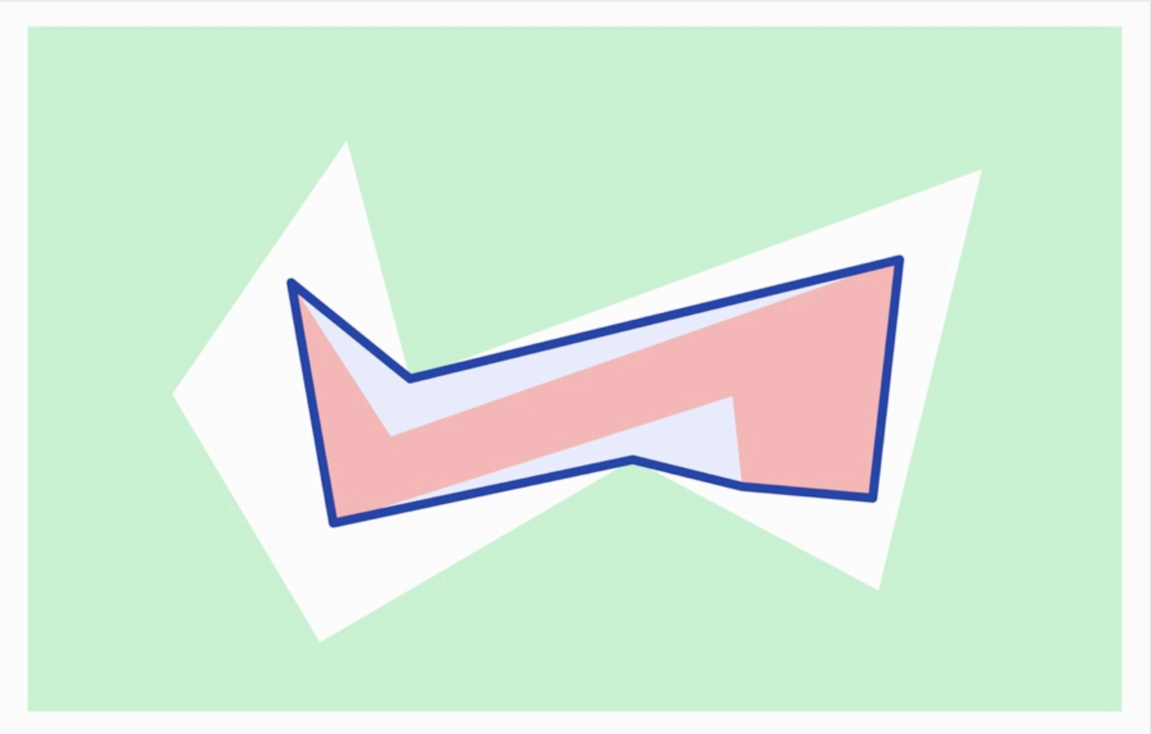
Relative Convex Hull

- The *convex set relative to G* contains every segment disjoint from *G* connecting some pair.
- The *convex hull of R relative to G* is the intersection of sets containing *R* relative to *G*.



(LEFT) NOT RELATIVELY CONVEX
(RIGHT) RELATIVELY CONVEX



RELATIVE CONVEX HULL

Prior Art: Relative Convex Hull

- SKLANSKY, J. AND KIBLER, D. 1976. *A theory of nonuniformly digitized pictures.* IEEE Transactions on Systems, Man, and Cybernetics 6, 637-647.
- SLOBODA, F. AND ZATKO, B. 2001. *On approximation of Jordan surfaces in 3D.* In Lecture Notes in Computer Science Volume 2243: Digital and Image Geometry, G. BERTRAND, A. IMIYA, AND R. KLETTE, Eds. Springer-Verlag, New York, NY, 365-386.
- Relative convex hull definition, properties, and computation.

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Motivating Tight Hulls

- The convex hull has widespread utility, including geometric simplicity.
- The relative convex hull generalizes the convex hull by excluding a set, but it is asymmetric.
- Seek a symmetric generalized convex hull to enhance its applicability.

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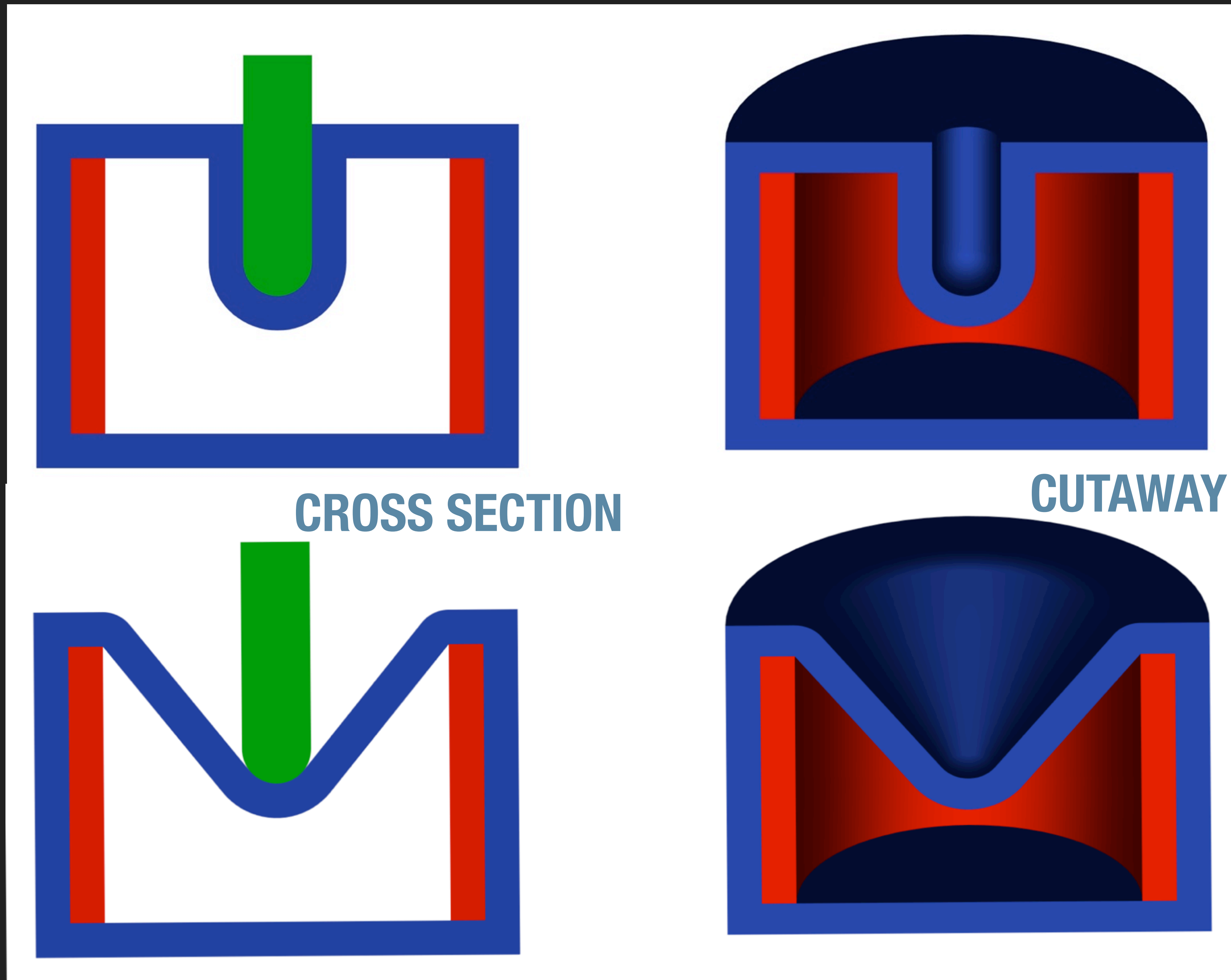

(ABOVE) RELATIVE CONVEX HULL IS ASYMMETRIC
(BELOW) TIGHT HULL IS SYMMETRIC

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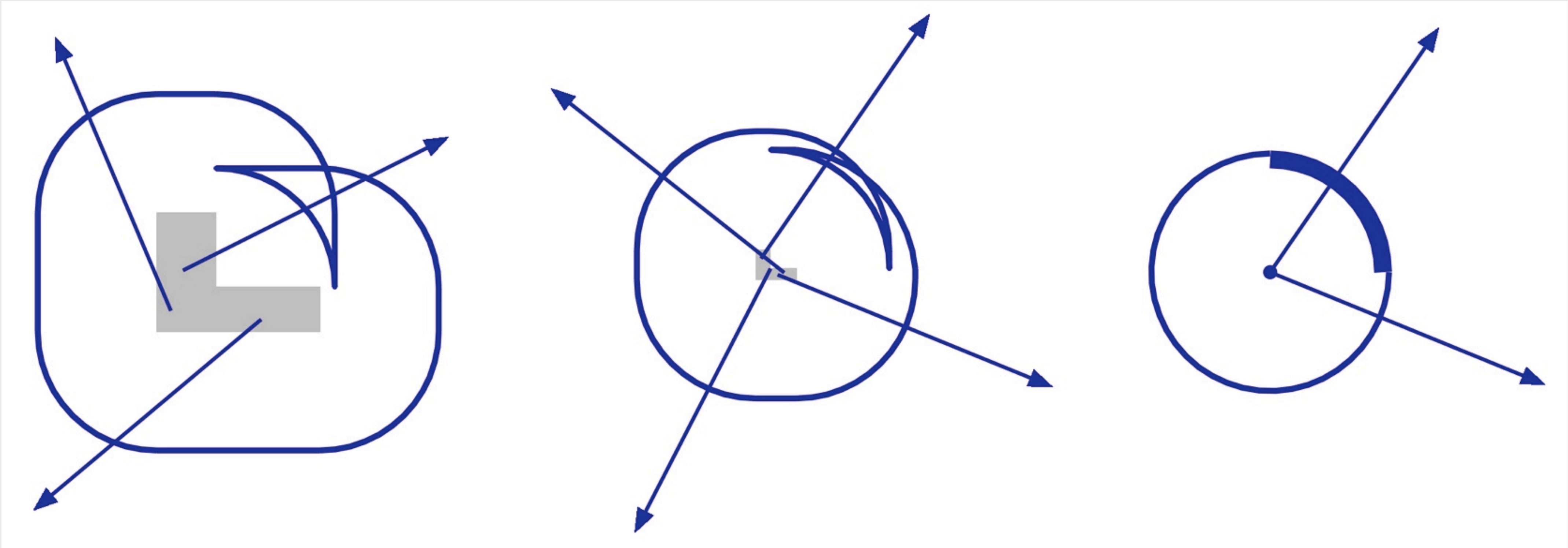
Tight Hull Definition

- Given disjoint sets *R* and *G*, consider candidates that contain *R* and exclude *G*.
- Reject candidates that do not minimize slack.
  - Slack generalizes total absolute Gaussian curvature.
- Return candidates that minimize unsupported slack.
  - Support quantifies relationship of hull to constraints.

Slack

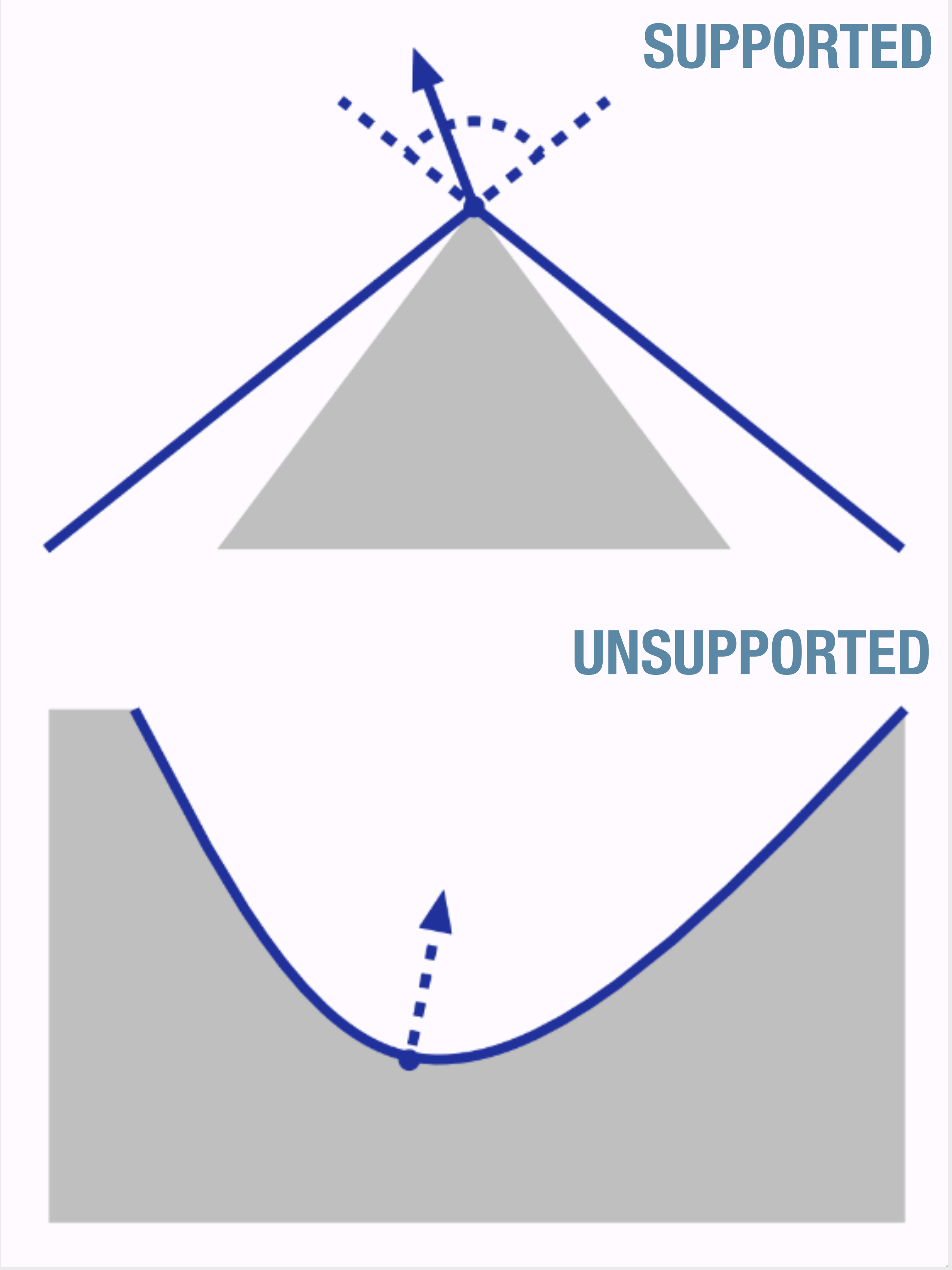
- Count the number of connected components with a given normal.
- Integrate the number of counts over all normals.

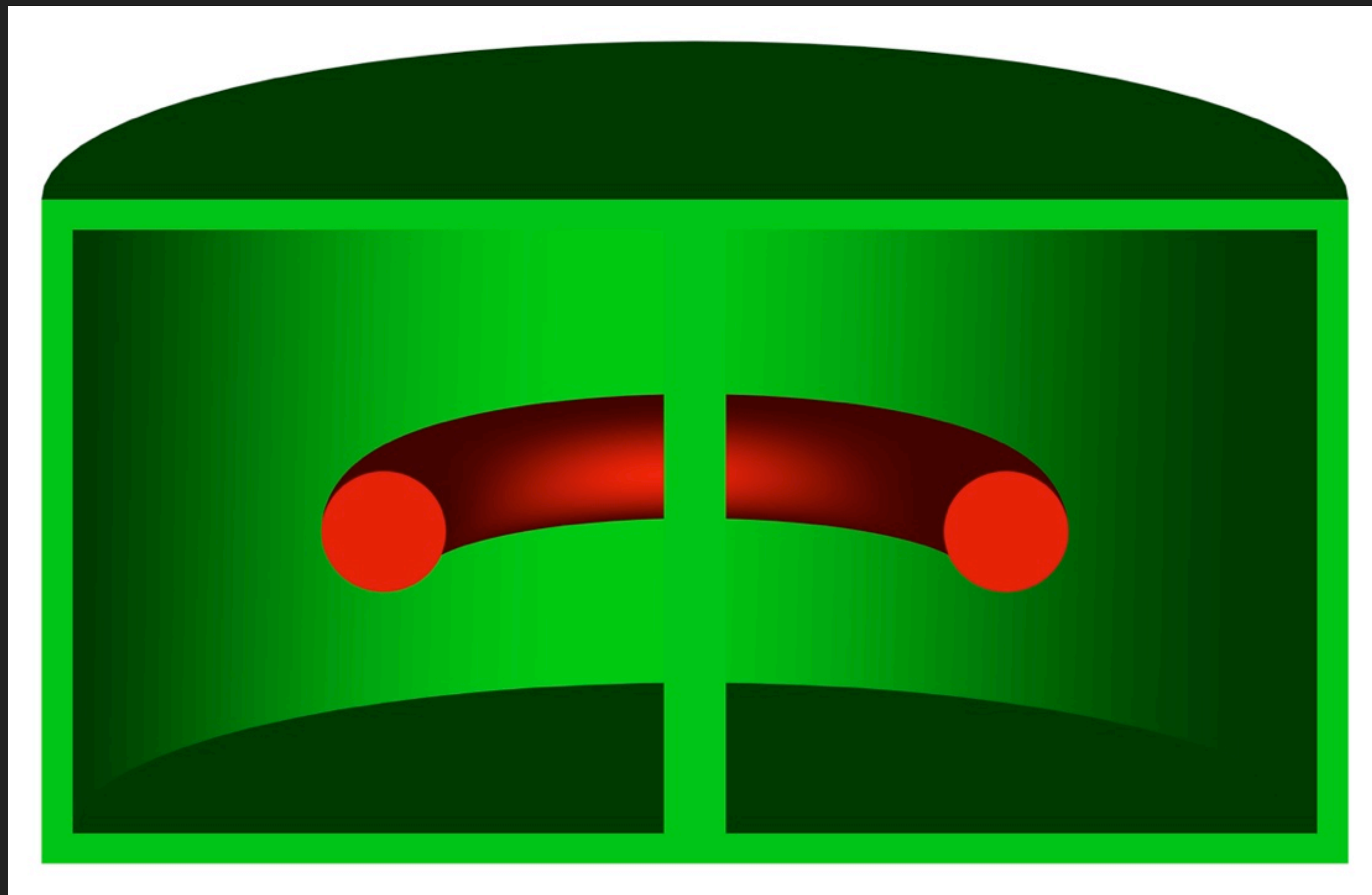
OFFSET OF BOUNDARY TO CIRCLE



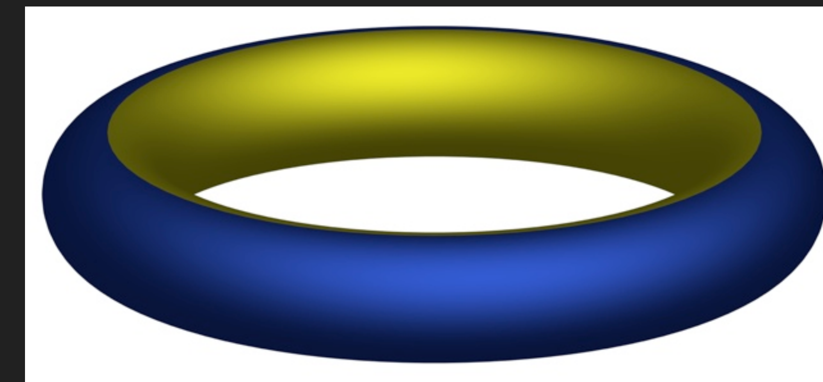
Support

- Convexity normal of *S* is the fraction of locally convex two-dimensional sections.
- Support of a convexity normal of *S* is its maximum dot product over the set of convex normals to *T*.

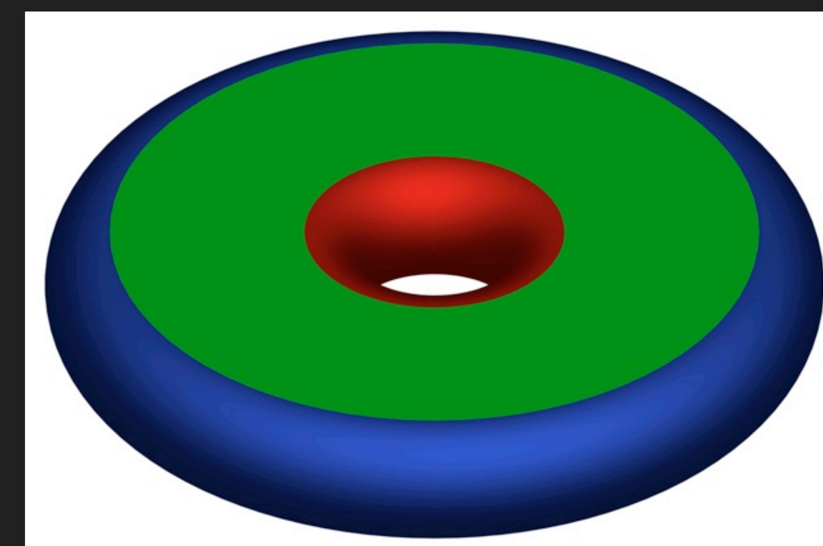


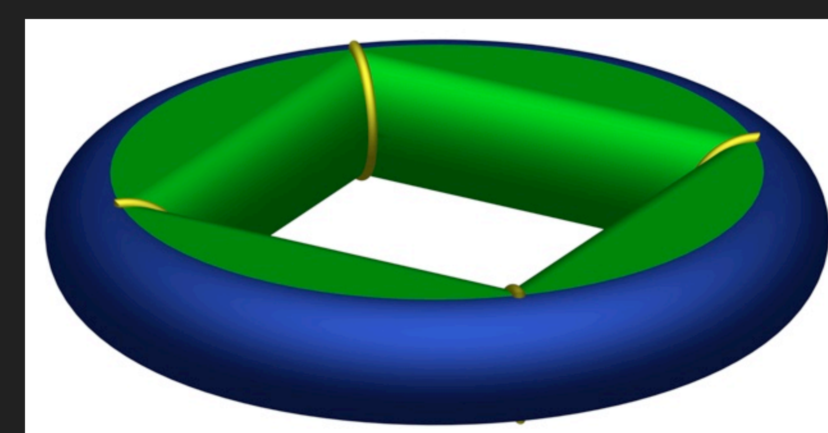


CUTAWAY INPUT



TIGHT HULL





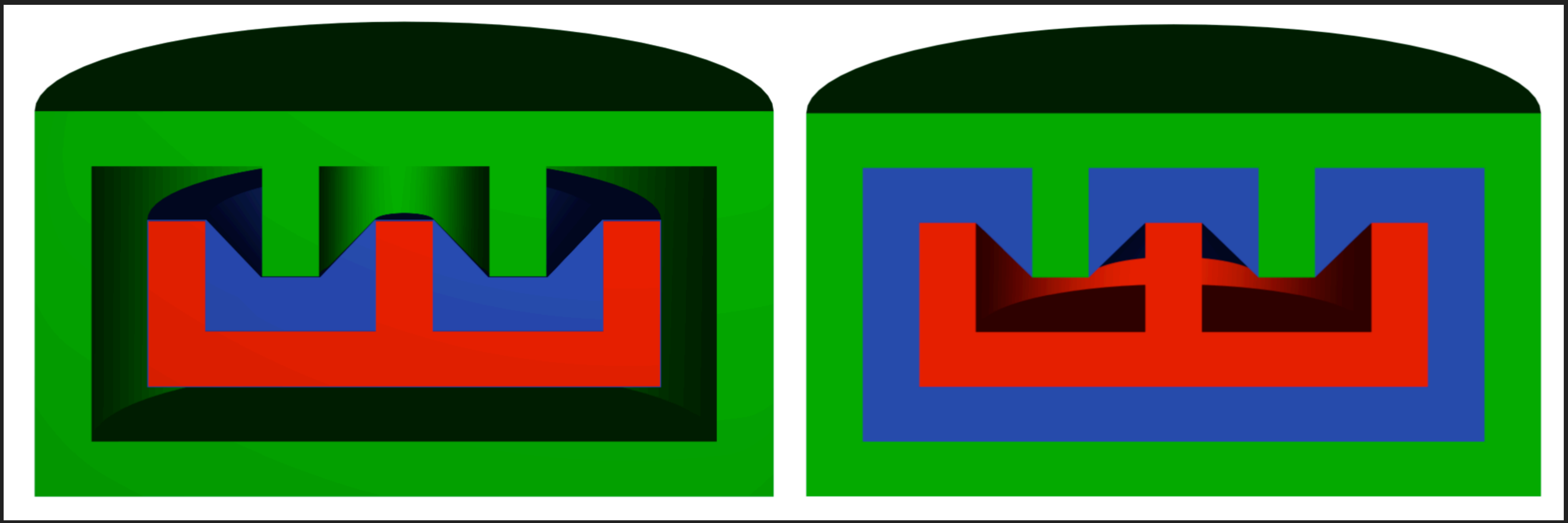
CANDIDATES NOT MINIMIZING UNSUPPORTED

TIGHT HULL MINIMIZES UNSUPPORTED SLACK

SUPPORTED (BLUE) PARTIALLY (YELLOW) UNSUPPORTED (RED) DEVELOPABLE (GREEN)

Tight Hull Desirable Properties

- The tight hull is symmetric.
- Minimizing slack eliminates boundary features.
- Minimizing unsupported slack reduces boundary measure.
- Qualitatively, the boundary is a membrane in tension.
- **Conjecture:** Unsupported boundary patches are developable, facilitating manufacture.



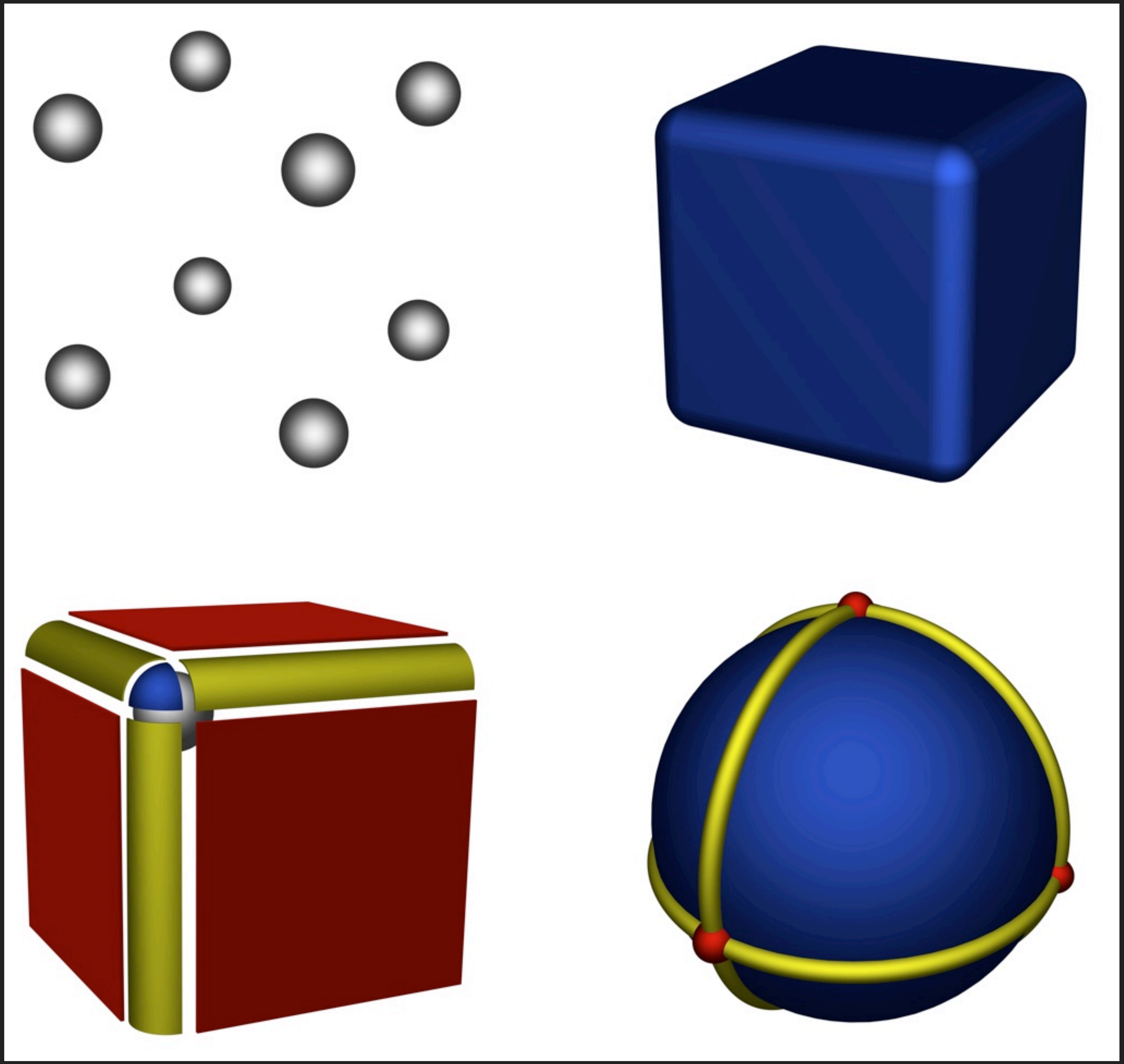
TH(R|G)

TH(G|R)

SYMMETRY

THE TIGHT HULL OF *R* RELATIVE TO *G* AND THE TIGHT HULL OF *G* RELATIVE TO *R*

INPUT



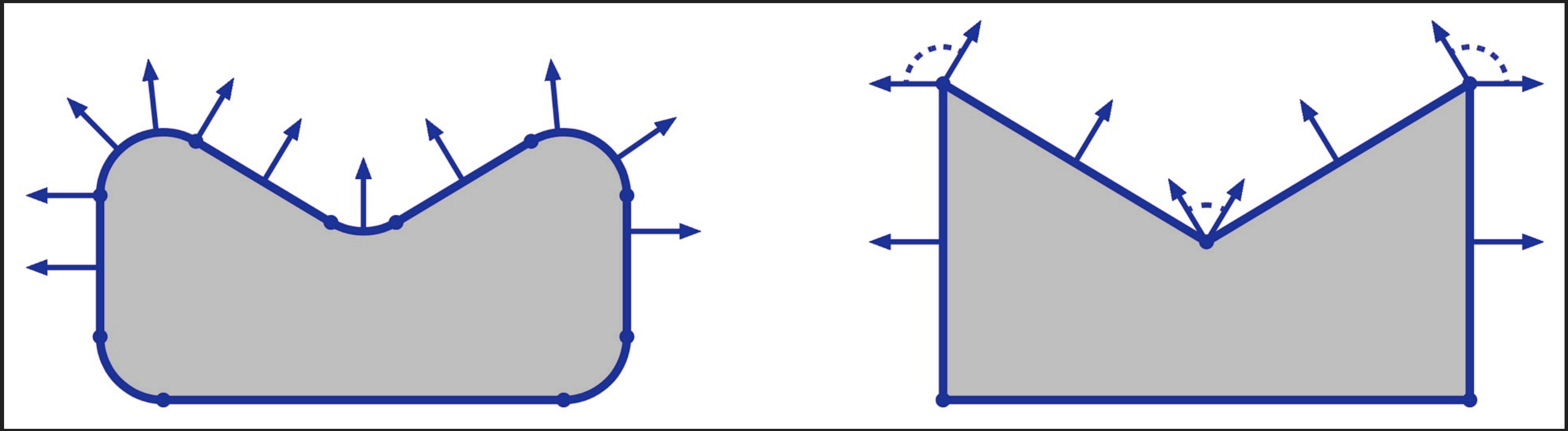
CONVEX HULL

CYLINDRICAL, PLANAR, SPHERICAL

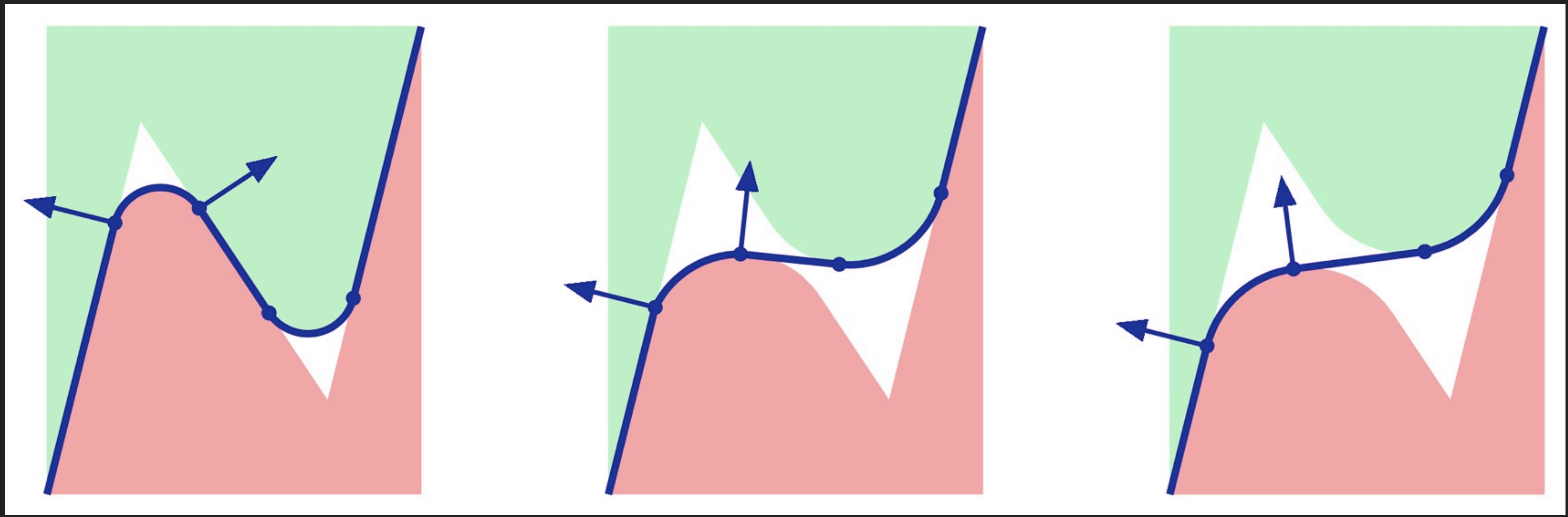
GAUSS MAP

DEVELOPABLE BOUNDARY

THE CONVEX HULL'S CYLINDRICAL AND SQUARE BOUNDARY PATCHES ARE DEVELOPABLE.



RADIUS INCREASED TO LEFT



RADIUS INCREASES LEFT TO RIGHT

UNPUBLISHED: TIGHT BLEND

OPENING INPUT INTERIOR AND COMPLEMENT PROGRESSIVELY SIMPLIFIES NORMAL FIELD.

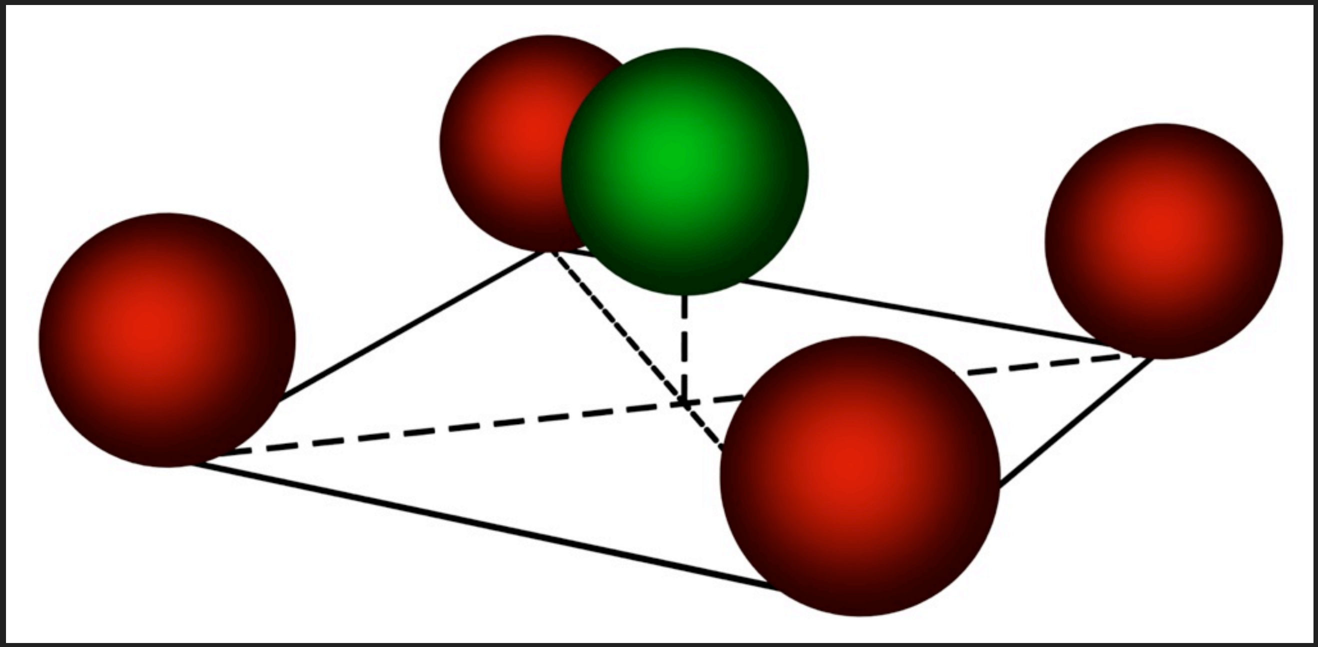
Motivating Tight Blends

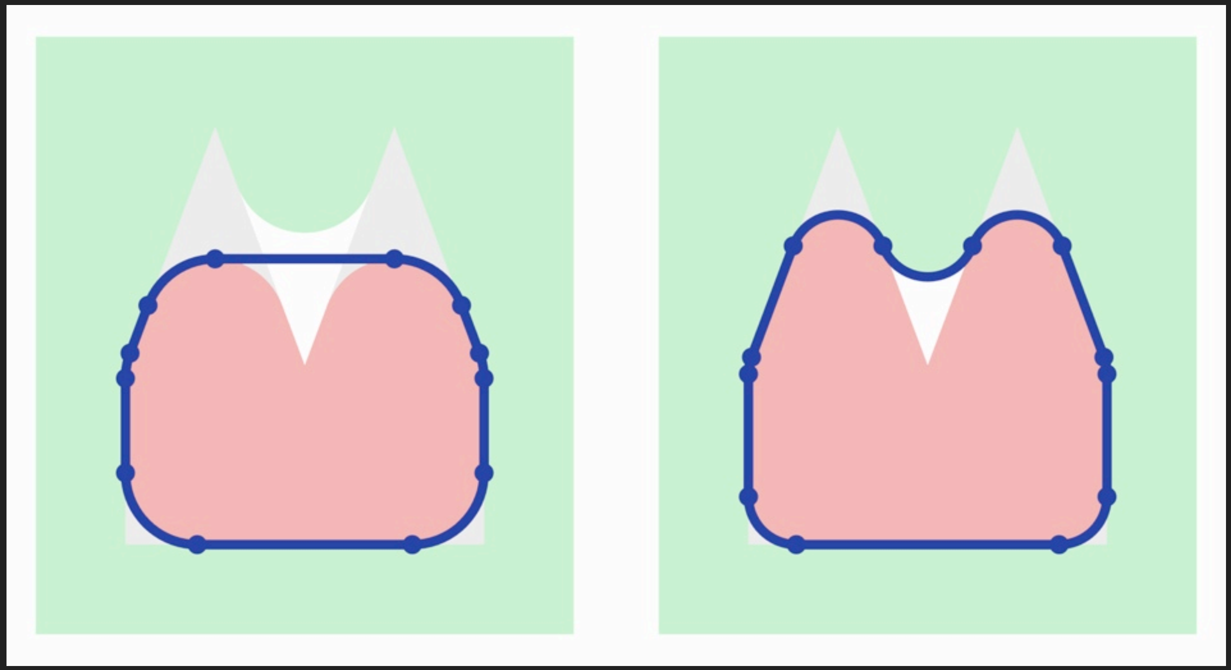
- Tight blends resemble tightening, but reduce Gaussian curvature rather than mean curvature.
- Unsupported tightening patches are minimal and saddle-shaped, while unsupported tight blends appear to be developable.
  - Reduce normal variation and light interaction.
  - Facilitate manufacturing and reduce cost.
- Tight blends offer alternative surface properties for rounded mortar constraints.

Tight Blend Definition and Desirable Properties

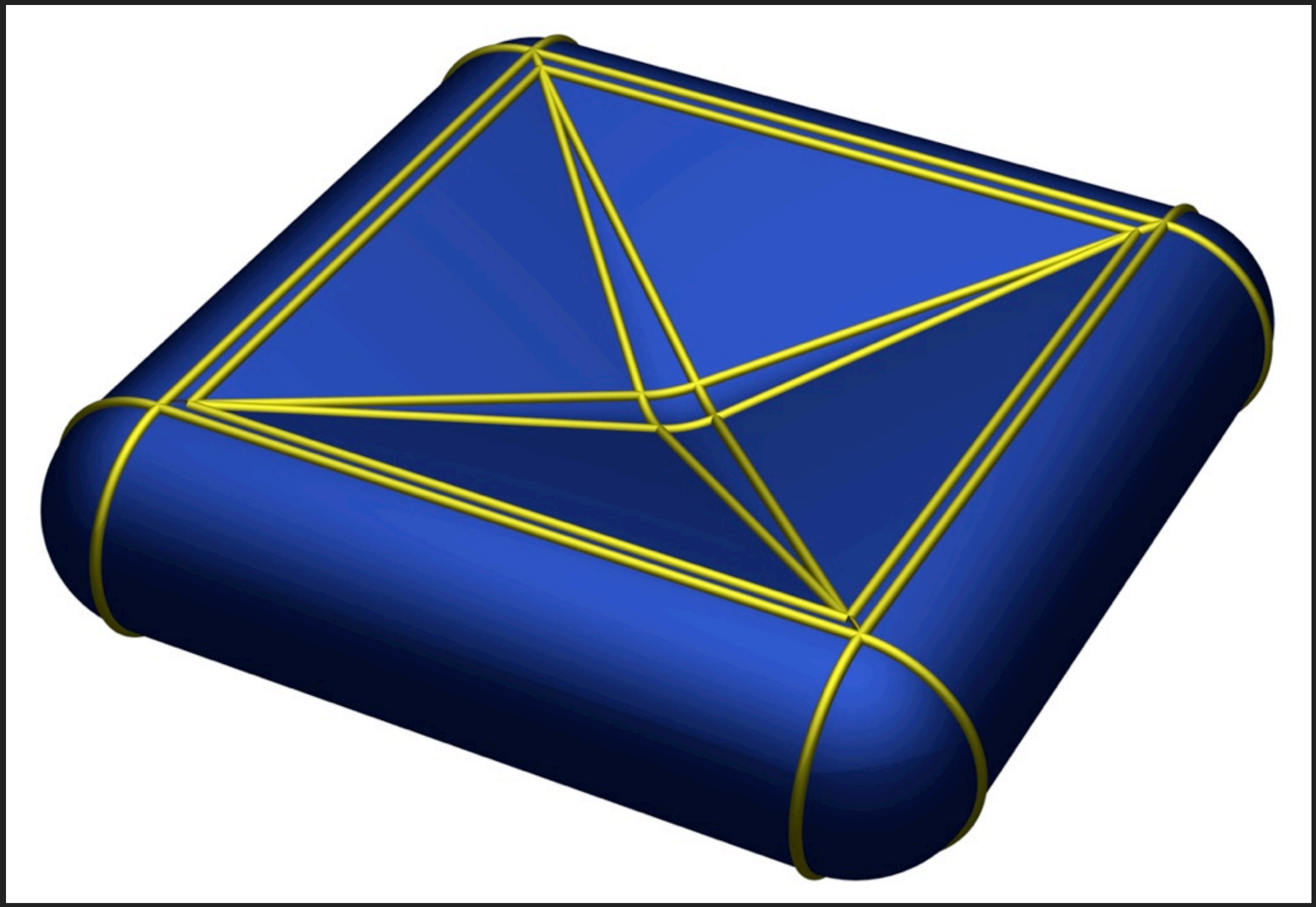
- A tight blend of a set is the tight hull of its core relative to its anticore.
- Tight blends share properties of tight hulls.
- Parameterized by radius, tight blends simplify normal fields.
- Qualitatively, normal field simplification simplifies shading.

RED BALLS CONTAINED,
GREEN EXCLUDED





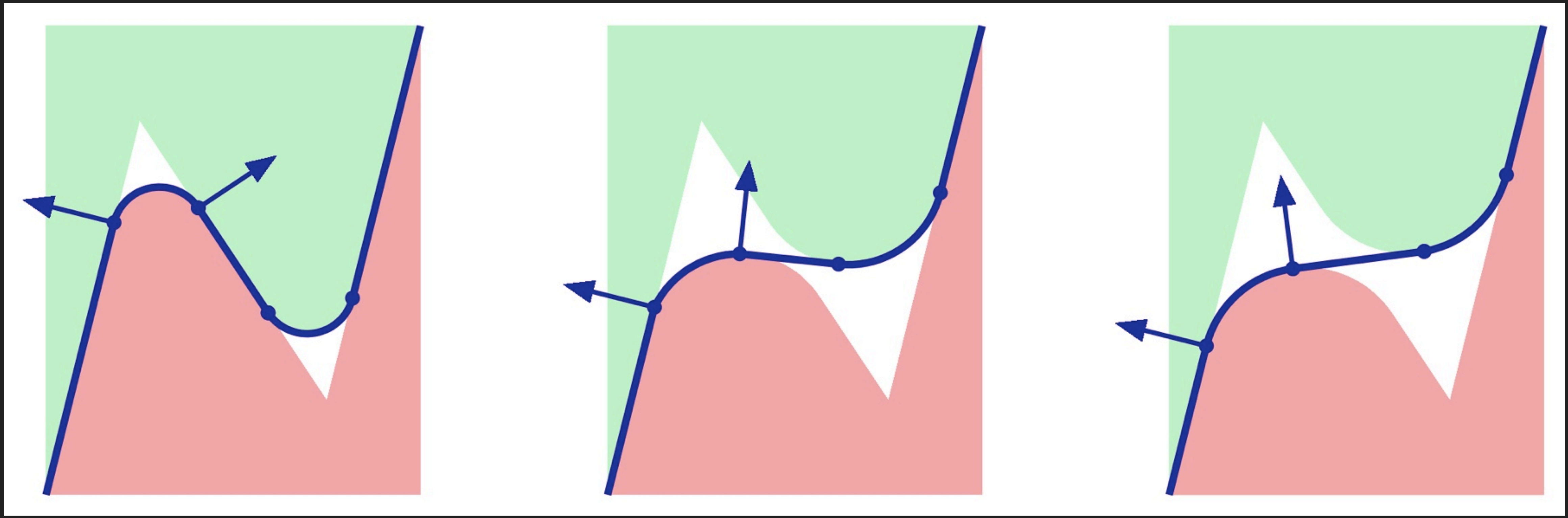
2D TIGHT BLEND, TWO RADII



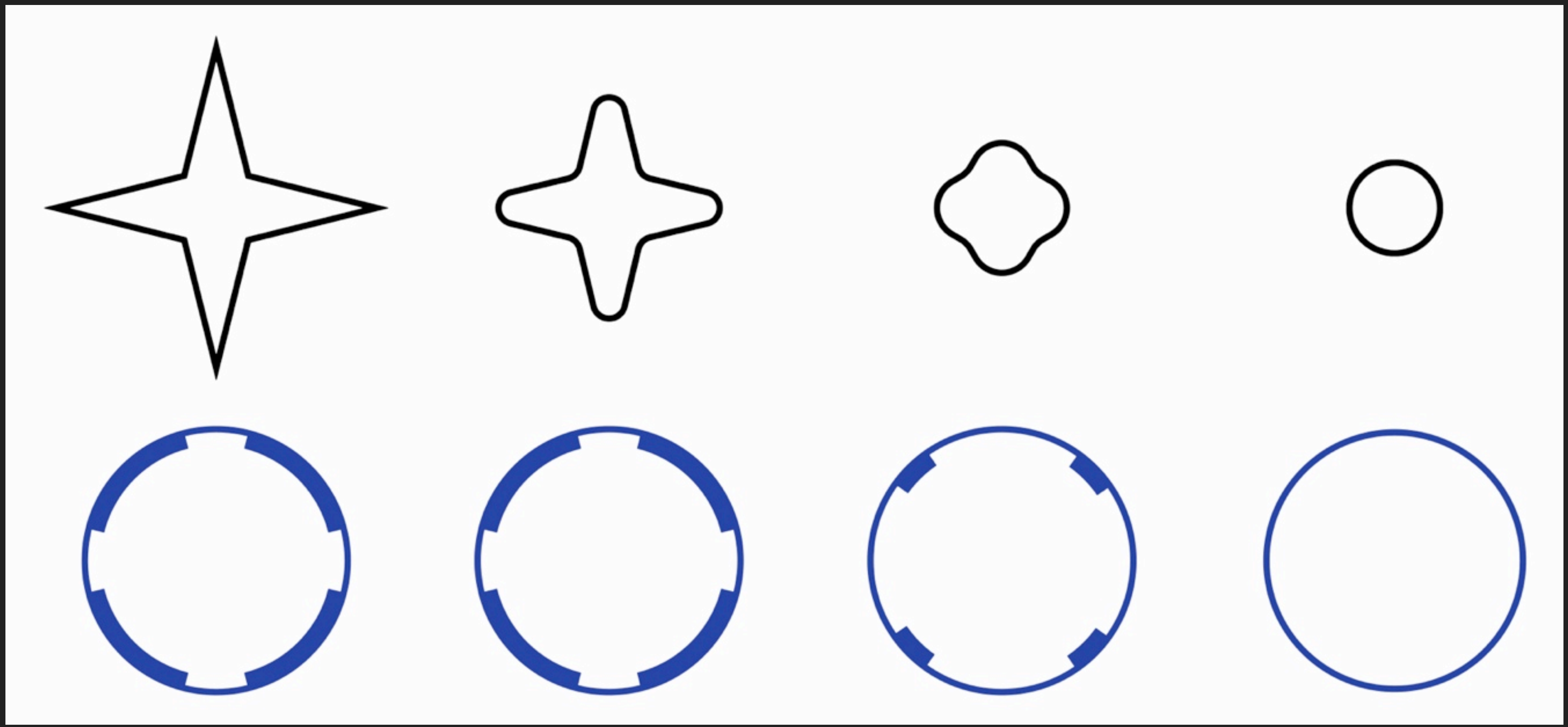
3D TIGHT BLEND,
YELLOW PATCH OUTLINES

TWO AND THREE DIMENSIONS

CONJECTURE: A BOUNDARY POINT LIES ON AN ARC OF RADIUS -1/*R* TO 1/*R*.



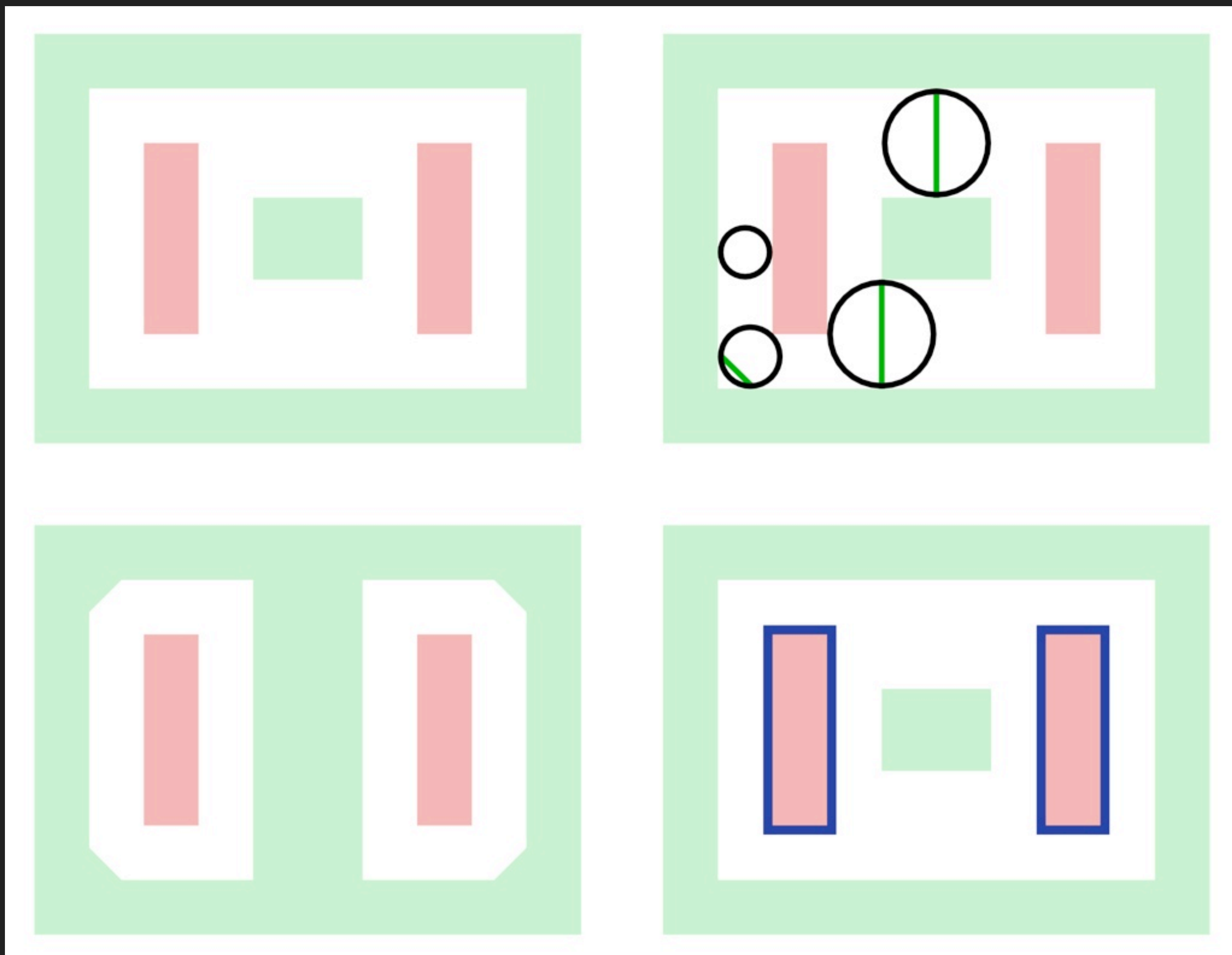
NORMAL CHANGE DECREASES LEFT TO RIGHT



INTEGRAL NORMAL COUNT DECREASES LEFT TO RIGHT

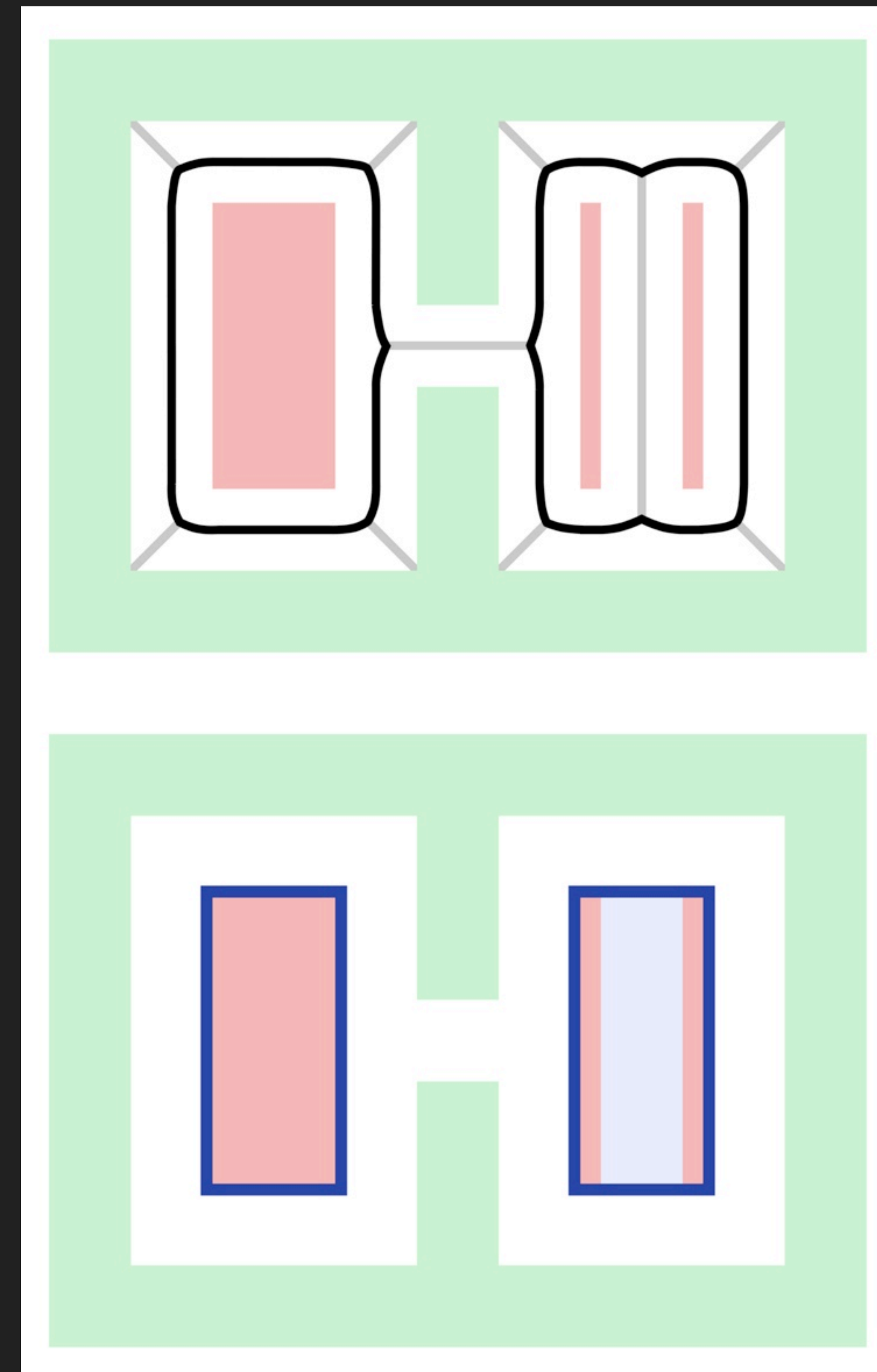
NORMAL FIELD SIMPLIFICATION

NORMAL CHANGE DECREASES AS OPENING RADIUS INCREASES WITH CONTINUOUS TOPOLOGY.



BOUNDARY IN ANNULI

ISOTOPIC MEDIAL AXIS EQUIDISTANT

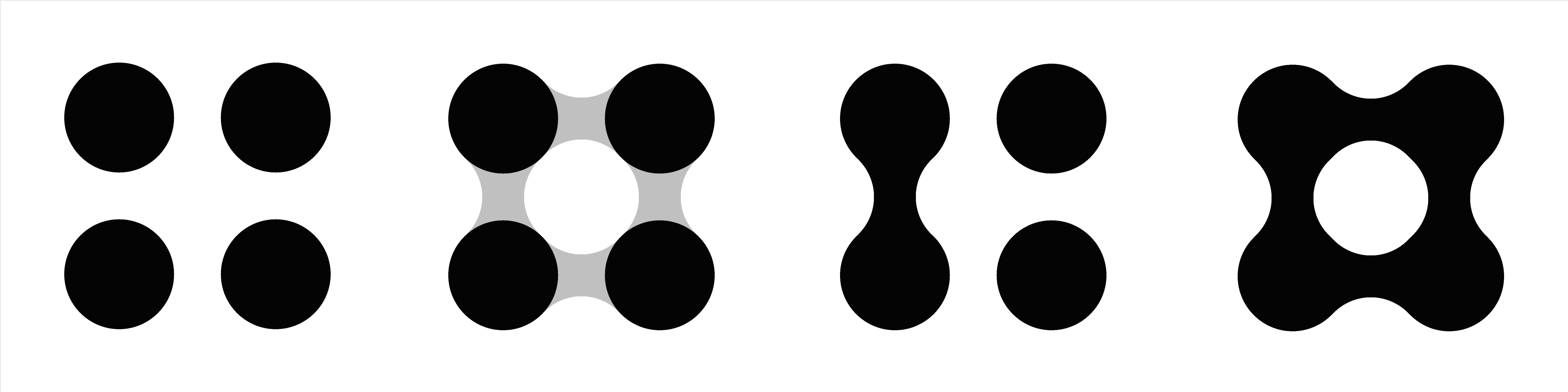


UNPUBLISHED: MEDIAL COVER

HULL BOUNDARY ISOTOPIC TO MEDIAL AXIS SUBSET EQUIDISTANT FROM *R* AND *G*

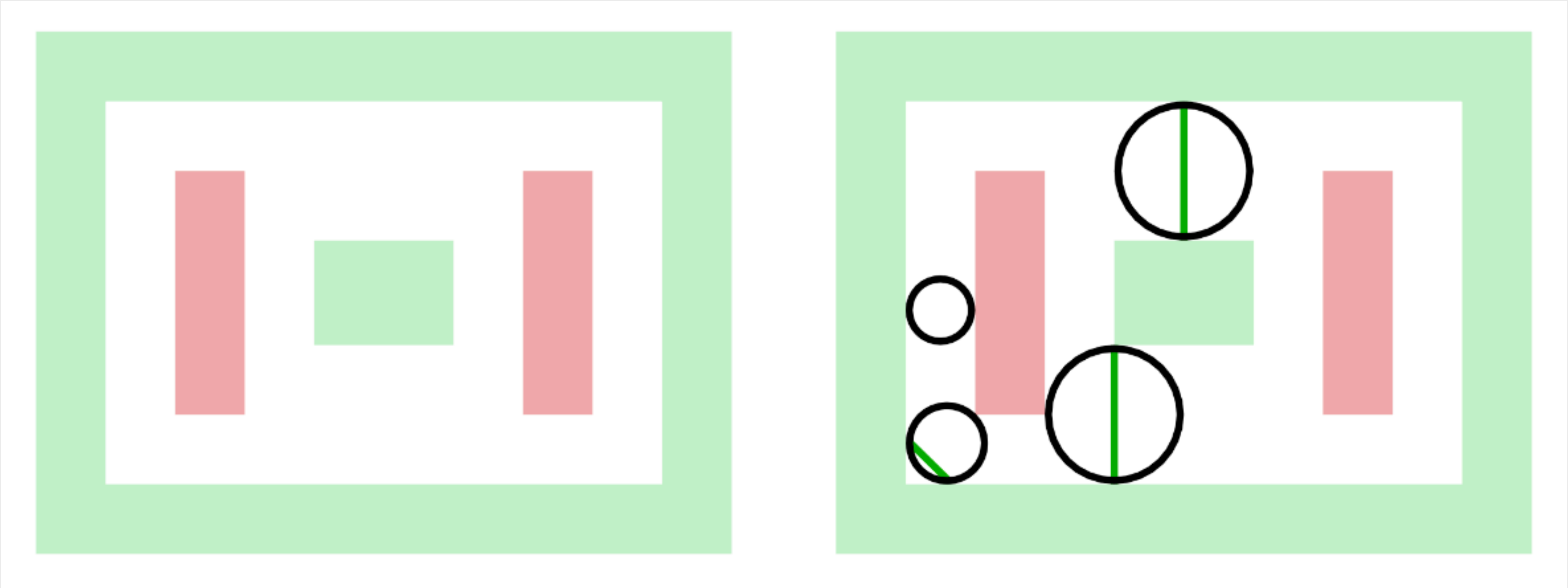
Motivating Medial Covers

- Tightening topology is generally nonunique.
- Tractable algorithms do not guarantee tight hull or tight blend topologies.
- We seek a tractable topology for 2D polygonal input with desirable properties.



Medial Cover Definition

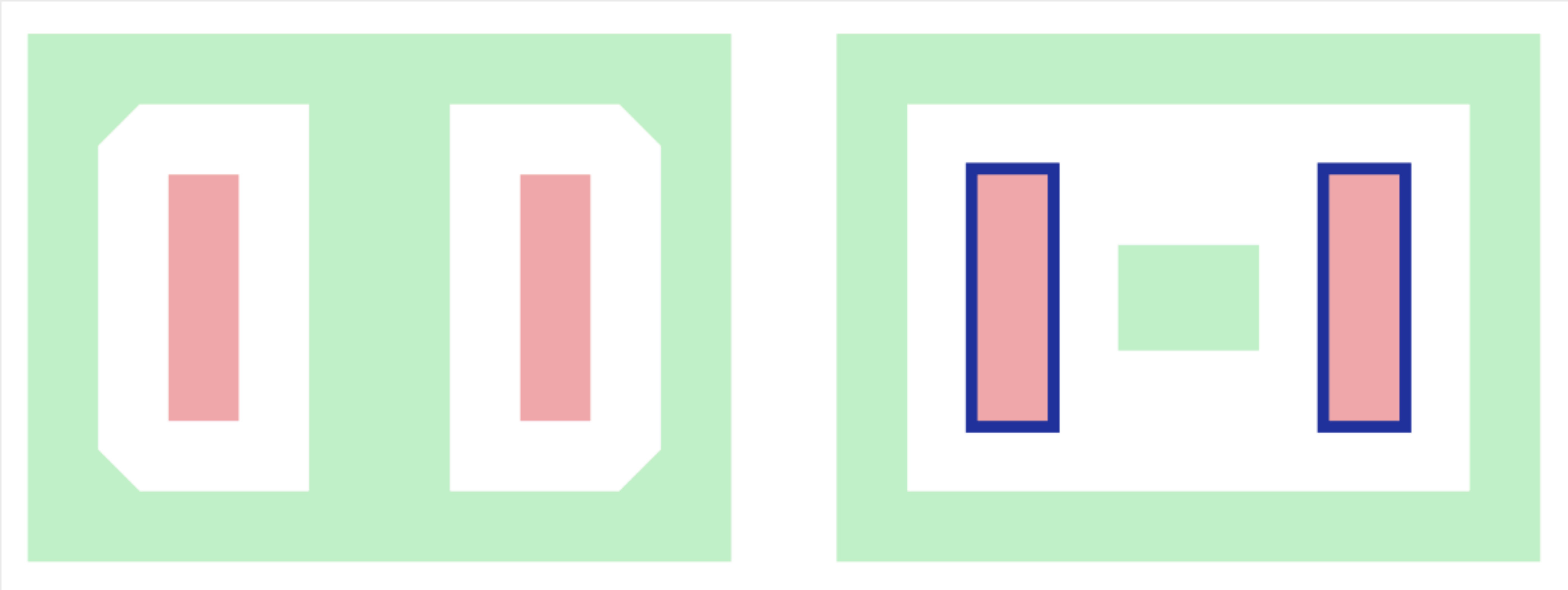
- Given 2D disjoint sets *R* and *G*,
  - Add to *R* convex hulls of maximal disk contact points with *R*.
  - Add to *G* convex hulls of contact points with *G*.



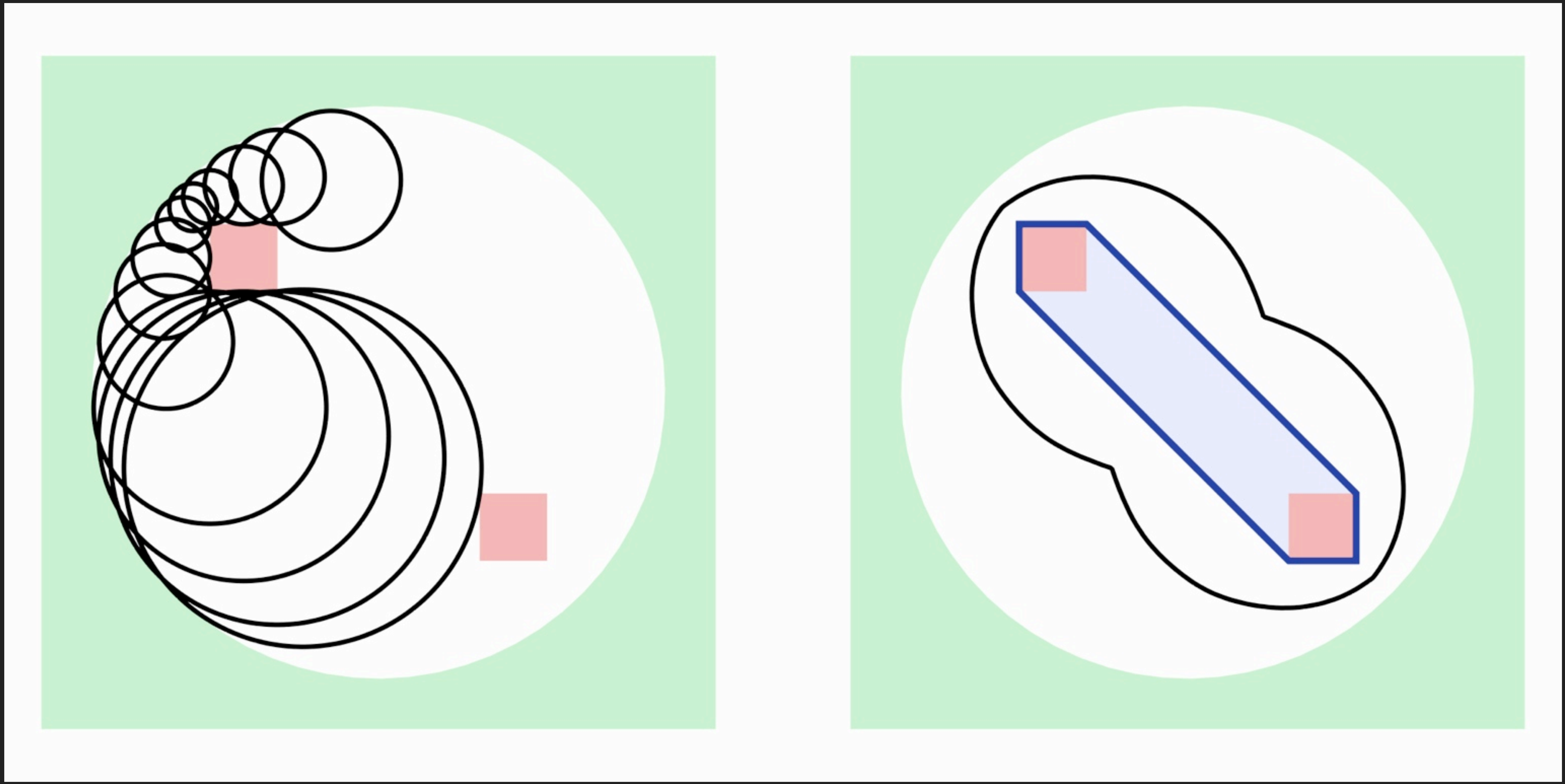
INPUT **ADD DARK GREEN SEGMENTS TO *G***

Medial Cover Definition

- Annuli separate *R* and *G*.
- Each annulus defines a tight hull loop.
- Medial cover boundary consists of tight hull loops.



ANNULI　　　　TIGHT HULL LOOPS



MAXIMAL DISKS

MEDIAL AXIS SUBSET, TIGHT HULL LOOP

MEDIAN CURVE (VORONOI BOUNDARY) ISOTOPY

CENTERS OF MAXIMAL DISKS EQUIDISTANT FROM *R* AND *G* ARE ISOTOPIC TO BOUNDARY.

Medial Axis Background

- CHOI, H., CHOI, S., AND MOON, H. 1997. *Mathematical theory of medial axis transform.* Pacific Journal of Mathematics 181, 57-88.

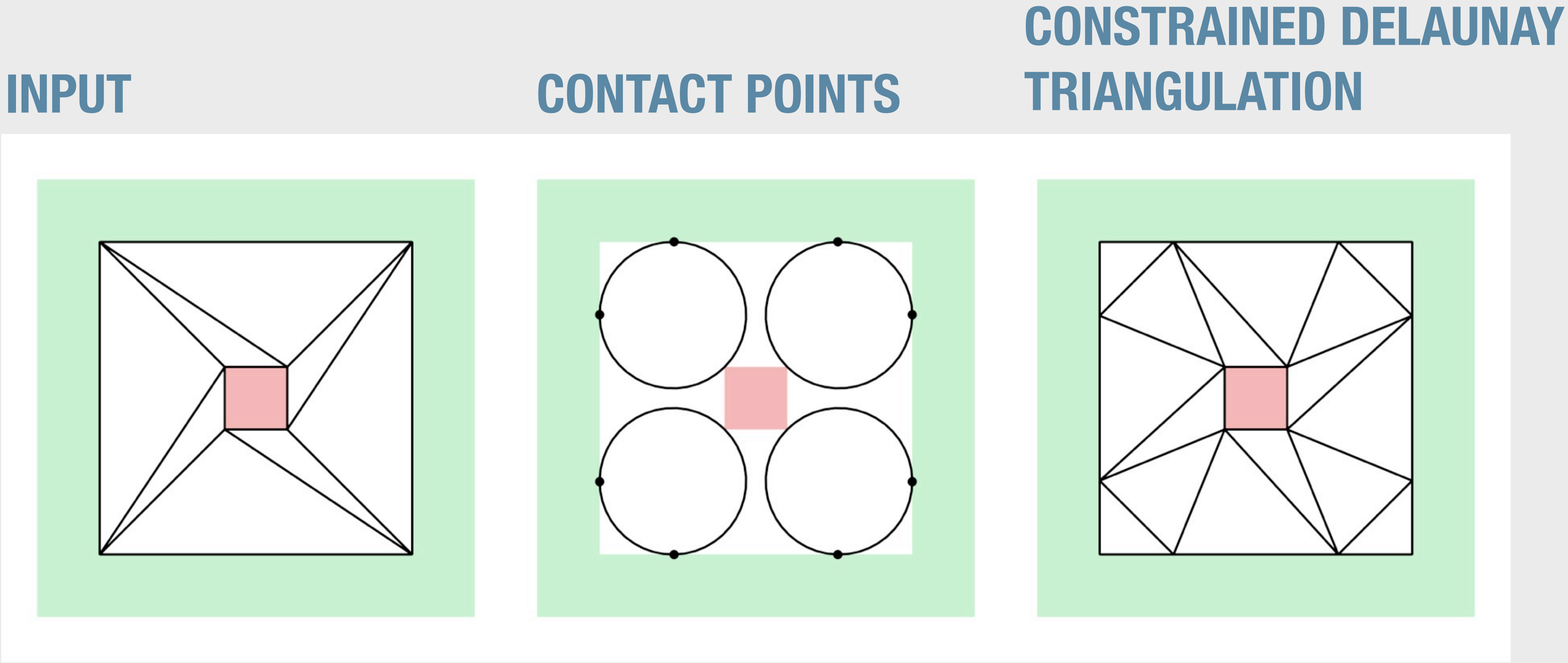
- PIZER, S., SIDDIQI, K., AND YUSHKEVICH, P. 2008. *Introduction.* In Computational Imaging 37: Medial Representations: Mathematics, Algorithms, and Representations, K. SIDDIQI AND S. PIZER, Eds. Springer, 1-34.

- WOLTER, F.-E. 1993. *Cut locus and medial axis in global shape interrogation and representation.* Design Laboratory Memorandum 92-2, Massachusetts Institute of Technology, Cambridge, MA.

- Medial axis theory.

Medial Cover Algorithm

- Given polygonal input, insert maximal disk bifurcation contact points.
- Constrained Delaunay triangulation of augmented vertices.

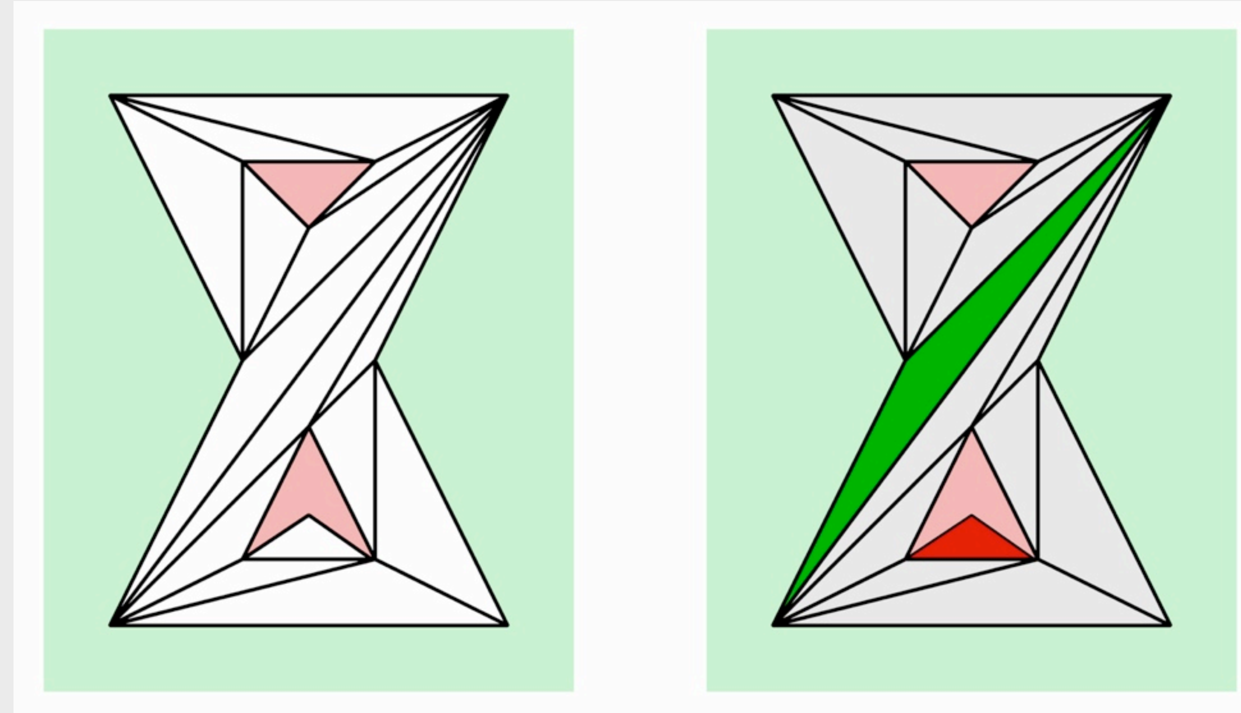


Prior Art: Constrained Delaunay Triangulation

- CHEW, L. 1987. *Constrained Delaunay triangulations.* Proceedings of the 3rd Annual Symposium on Computational Geometry, Waterloo, Canada, June 1987, ACM Press, 215-222.
- Constrained Delaunay triangulation.

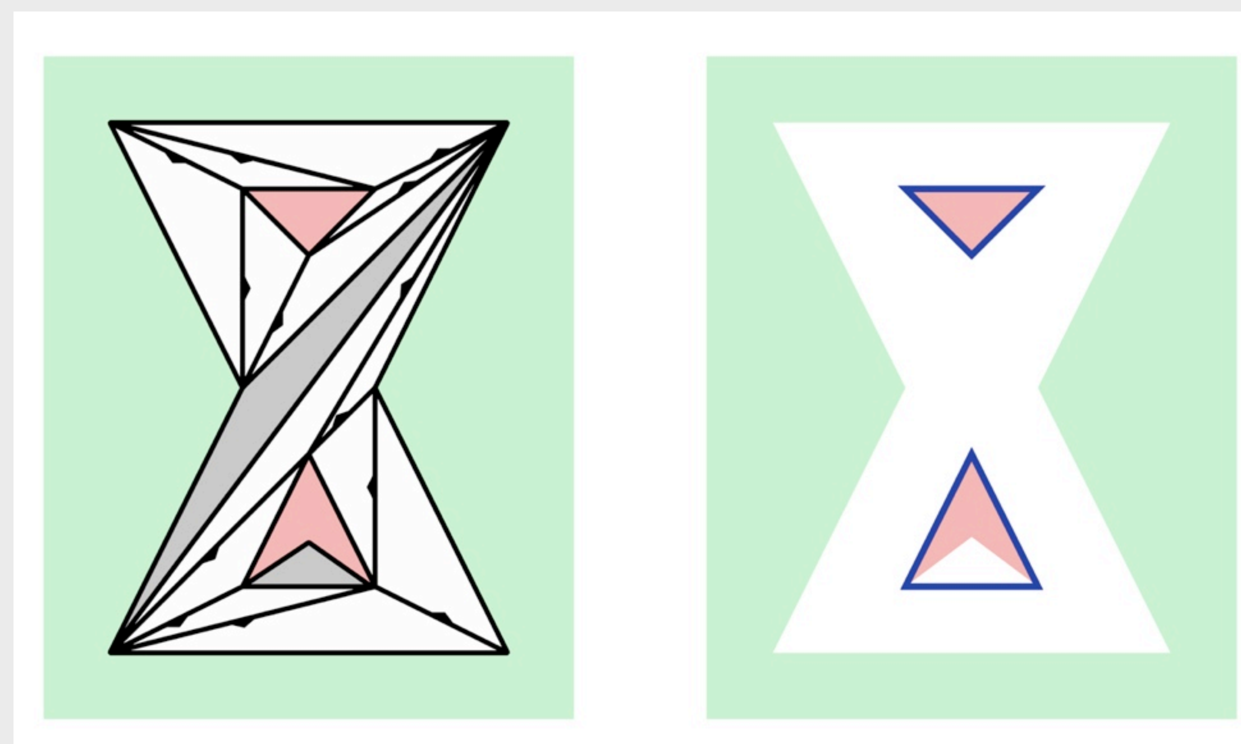
Medial Cover Algorithm

- Group triangles contacting both *R* and *G* to form annuli.
- Use polygonal path planning to compute a loop in each annulus.



TRIANGULATION

MIXED (GRAY)
SOLID (RED,GREEN)

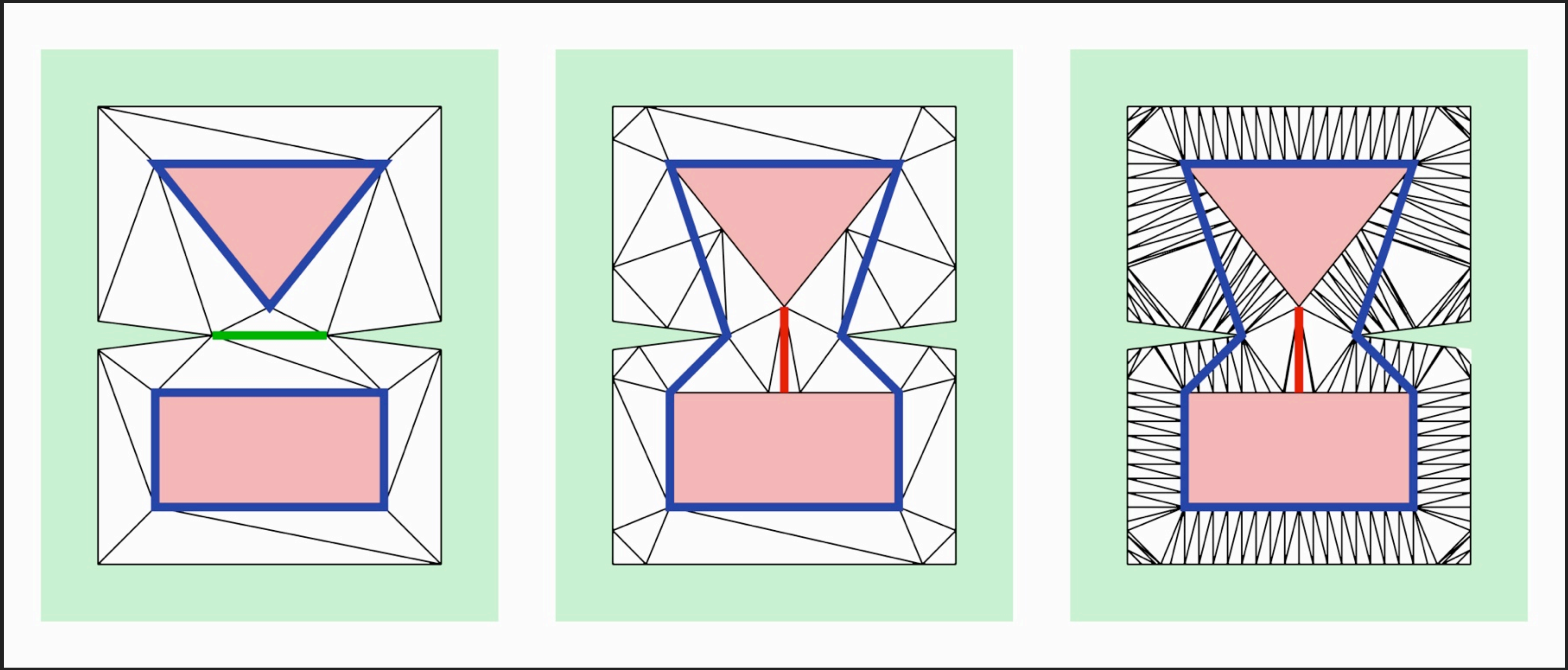


ANNULI

PATH PLANNING

Prior Art: Funnel Algorithm

- CHAZELLE, B. 1982. *A theorem on polygon cutting with applications.* Proceedings of the 23rd Annual Symposium on Foundations of Computer Science, Chicago, IL, November 1982, IEEE Computer Society, New York, NY, 339-349.
- LEE, D. AND PREPARATA, F. 1984. *Euclidean shortest paths in the presence of rectilinear barriers.* Networks 14, 393-410.
- Linear time algorithm for shortest path planning in simple polygons.



INPUT SAMPLES BIFURCATION SAMPLES ADDED SAMPLES

ADDING EDGE SAMPLES

ONCE BIFURCATION SAMPLES ARE INTRODUCED, ADDING SAMPLES DOES NOT CHANGE TOPOLOGY

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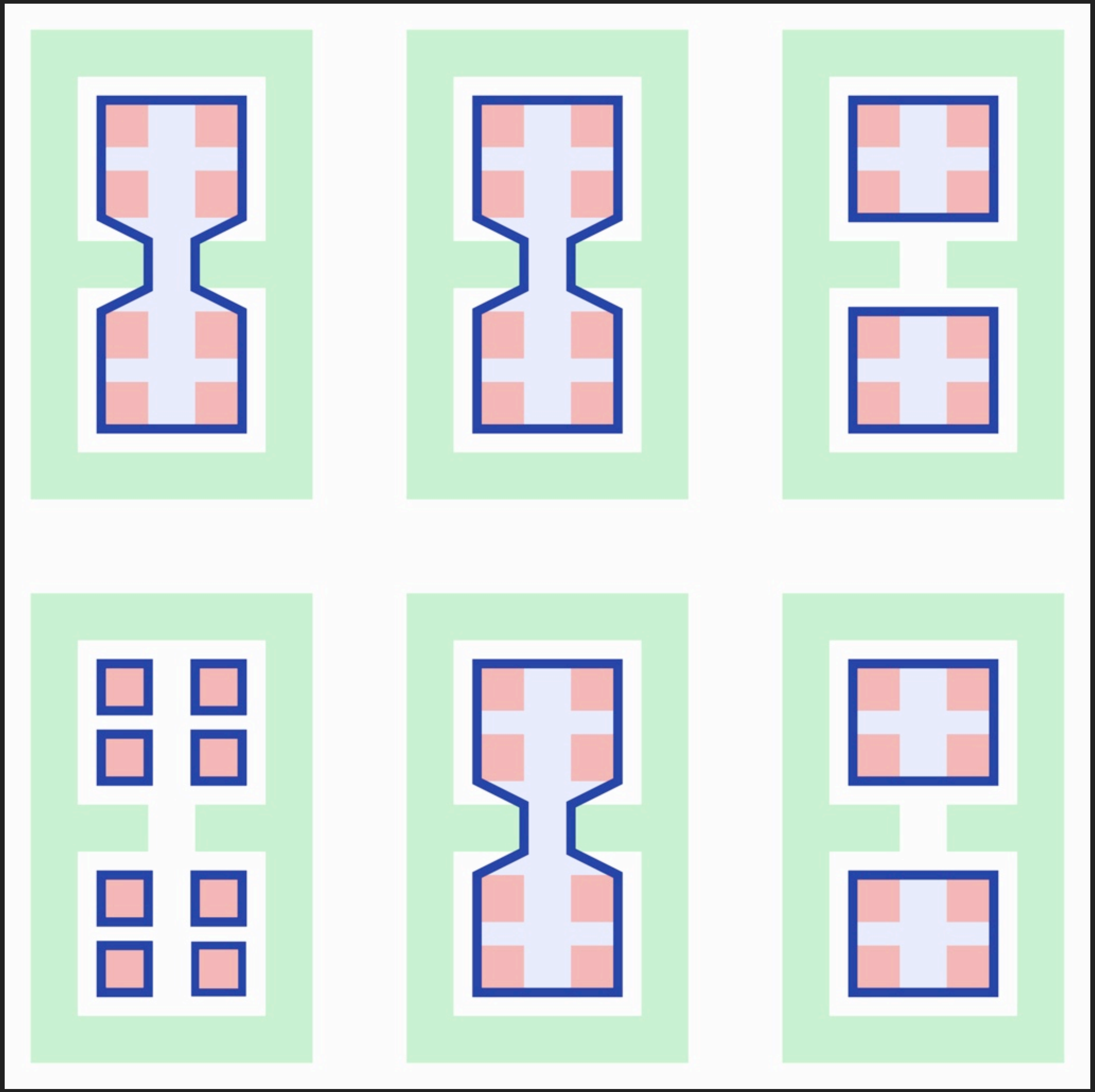
Medial Cover: Desirable Properties

- Medial cover is symmetric.
- Medial cover loops organize components by proximity.
- Computing the medial axis and constrained Delaunay triangulation requires O($n$log n) time, while the remainder is linear.
- Implementation composes established, reliable, efficient geometric algorithms.

RCH(R|G)

RCH(G|R)^C

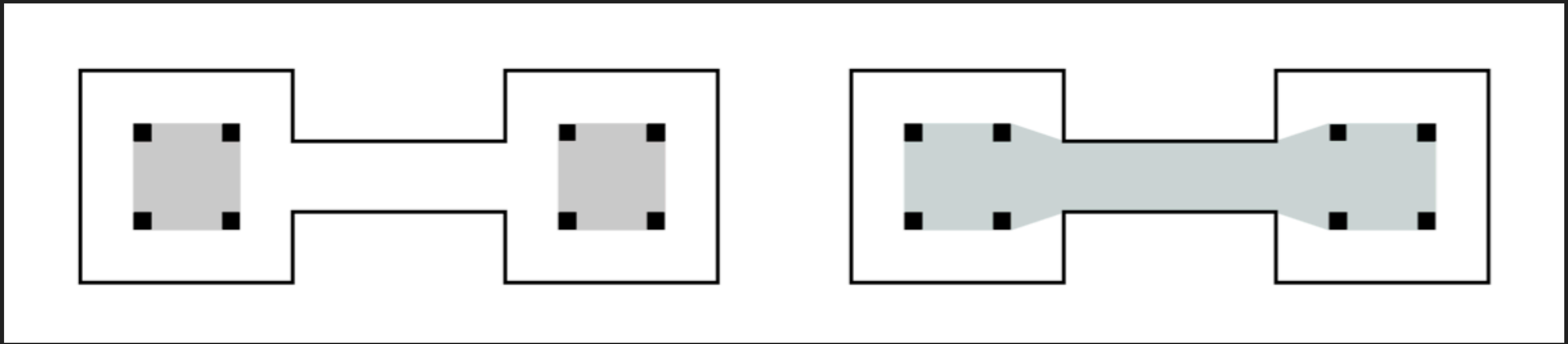


TIGHT HULL (CENTER)

MEDIAL COVER (RIGHT)

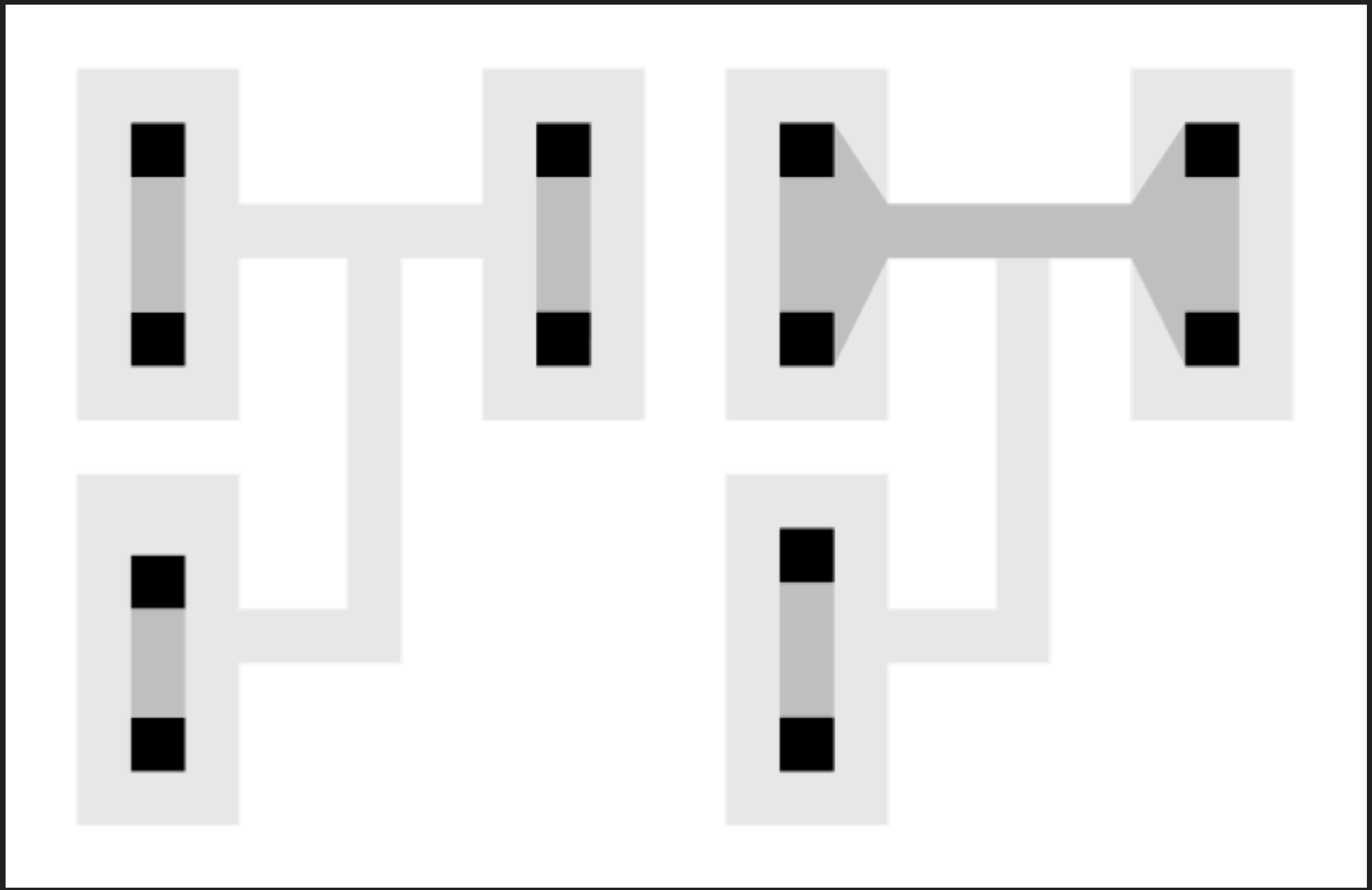
SYMMETRY

RCH(R|G) AND RCH(G|R)^C ARE DIFFERENT, WHILE TIGHT HULL AND MEDIAL COVER ARE NOT.



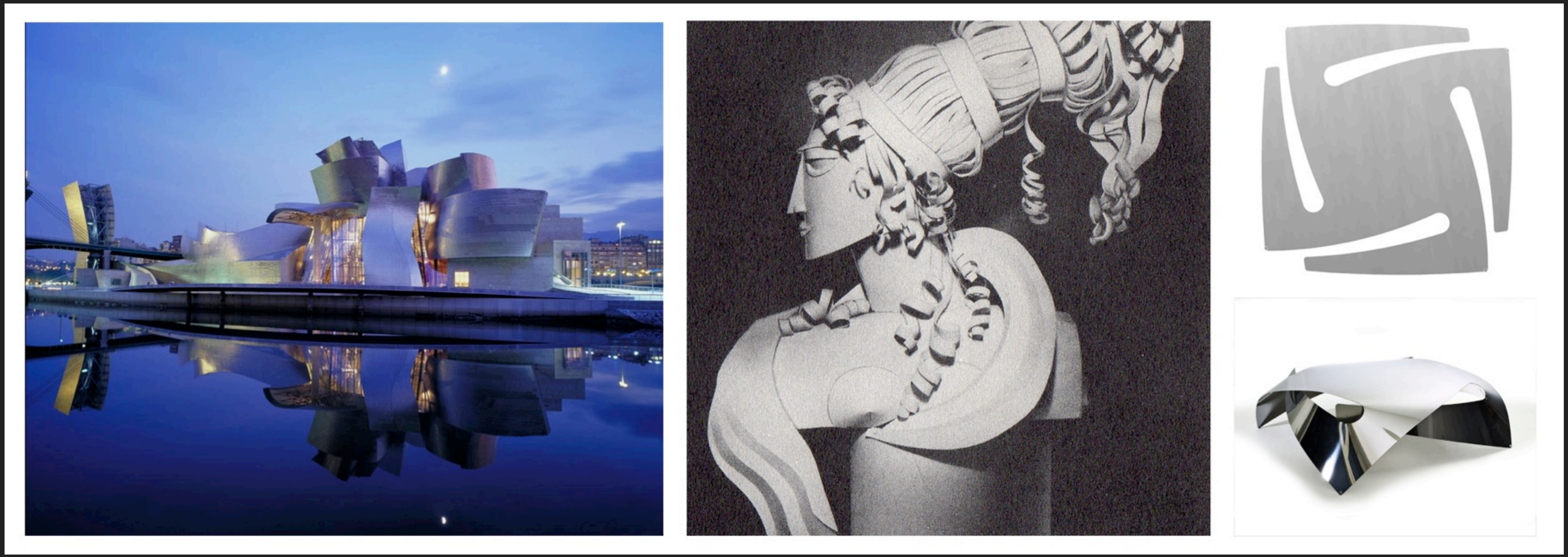
MEDIAL COVER
(LEFT)

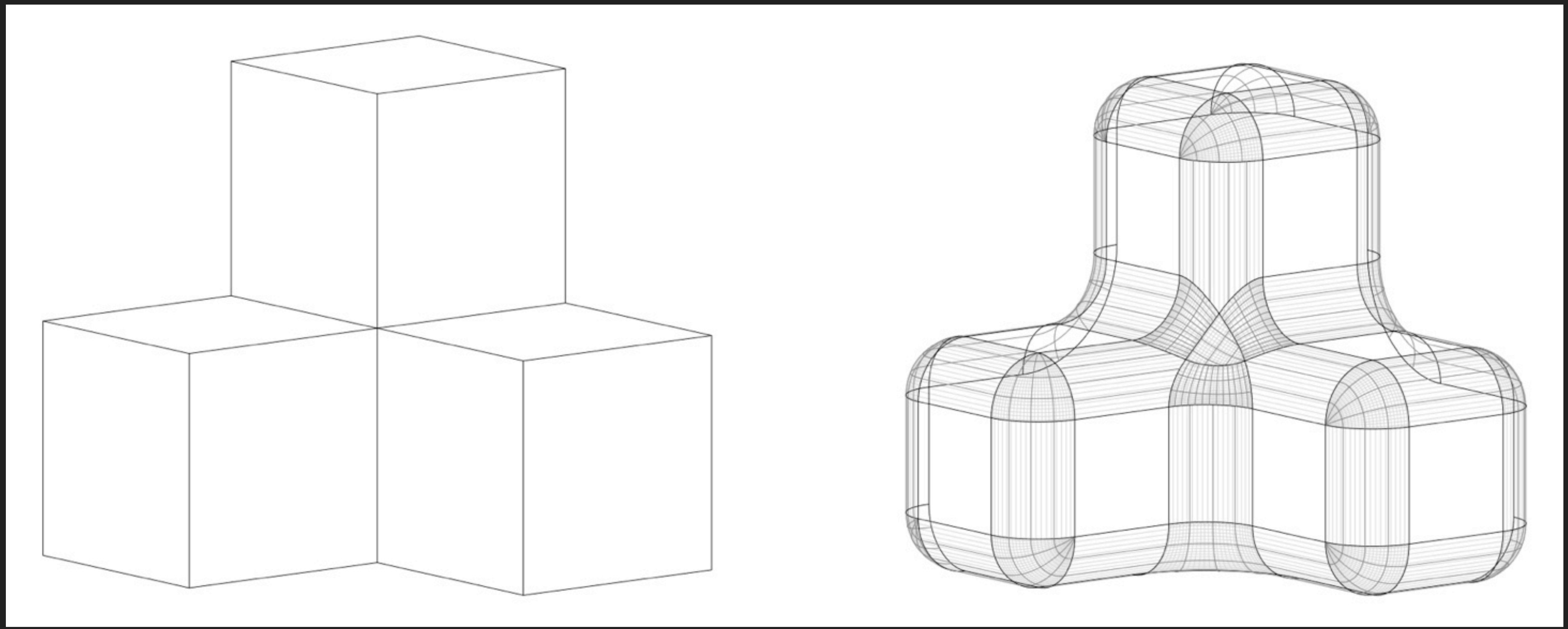
RELATIVE CONVEX HULL
(RIGHT)



PROXIMITY

THE MEDIAL COVER SELECTIVELY COLLECTS NEARBY COMPONENTS.





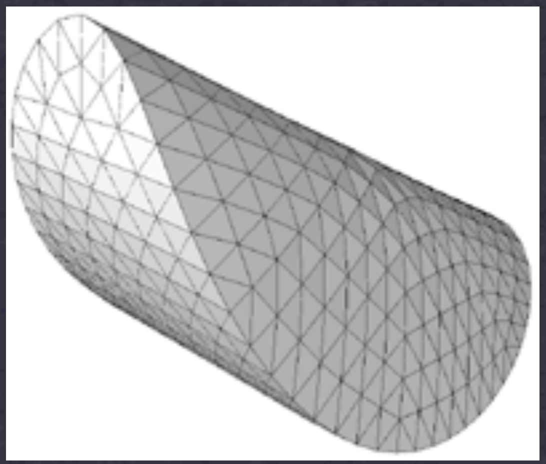
APPLICATIONS: TIGHTENING AND BLENDING

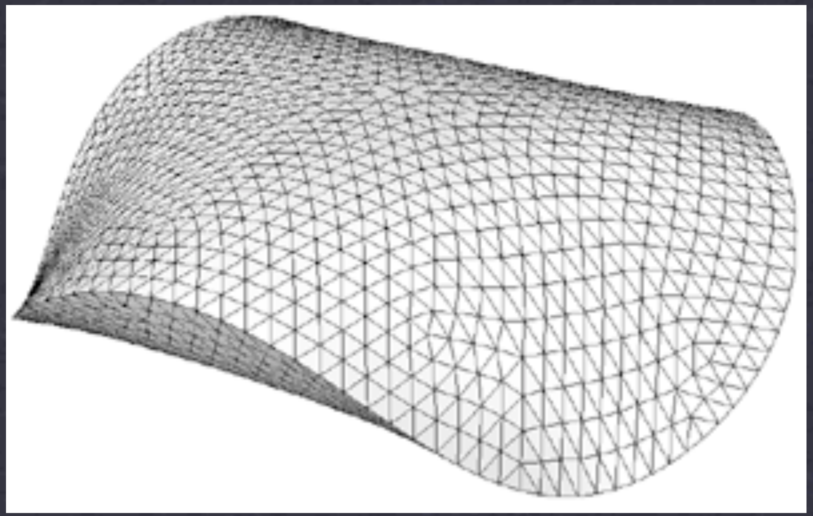
SOLID MODEL BOUNDARY OPTIMIZATION USING TIGHTENING AND BLENDING

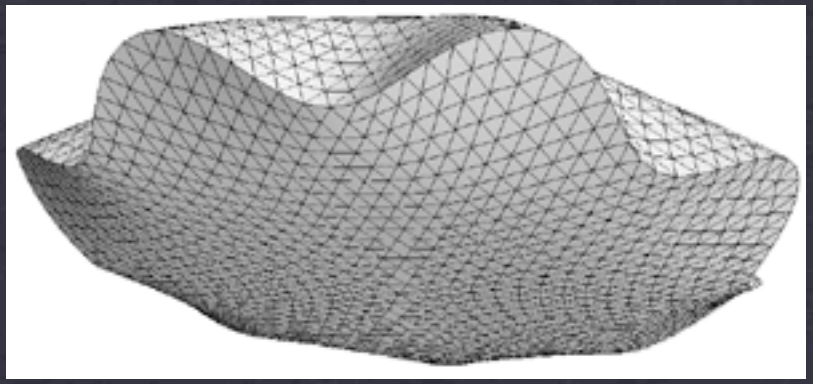
Apply: Tighten and Blend

- GORECKA-EGAN, E. Helen of Troy. In Paper Sculpture, by MCPHARLIN, P., 1944, Marquardt & Company, New York, NY, pp. 46.
- KADUSHIN, R. 2006. Square dance coffee table. http://ronen-kadushin.com/Open_Design.asp.
- SCARINCI, I. 2008. *Financial scandal at the Guggenheim Bilbao.* Translated by MAHABIR, A. ARCADJA artMagazine, http://www.arcadja.com.
- ROSSIGNAC, J. 1985. *Blending and Offsetting Solid Models.* Doctoral dissertation, University of Rochester, Rochester, NY.

D-FORM, WILLS







GENERALIZATION, BOURKE

APPLICATION: CONVEXITY

CONVEXITY IN RELATION TO TIGHTENING AND BLENDING

Prior Art: Convexity

- WILLS, T. D-forms. http://www.curvedfolding.com/
- BOURKE, P. d-Forms. http://paulbourke.net/geometry/dform/
- SHARP, J. 2009. *D-forms: Surprising New 3-d Forms from Flat Curved Shapes.* Tarquin Publications.
- POTTMANN, H., ASPERI, A., HOFER, M., AND KILIAN, A. 2007. *Architectural Geometry.* Bentley Institute Press, Exton, PA.

- Two convex shapes of equal boundary length cut from flexible sheets and attached along boundary.

Prior Art: Alternating Sequential Filter

- STERNBERG, S. 1986. *Grayscale morphology.* Computer Vision, Graphics, and Image Understanding, 333-355.
- Alternating morphological opening and closing with progressively increasing radius for smoothing and denoising.

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Prior Art: Mesh Signal Processing

- TAUBIN, G. 1995. *A signal processing approach to fair surface design.* Proceedings of the 22nd Annual Conference on Computer Graphics and Interactive Techniques, ACM, New York, NY, 351-358.
- Smoothing and denoising by alternating a discrete Laplacian operator.

Prior Art: Polygonal Repair

- HE,T., HONG, L., VARSHNEY, A., AND WANG, S. 1996. *Controlled Topology Simplification.* IEEE Transactions on Visualization and Computer Graphics 2, 171-184.
- Triangle mesh extracted from low-pass filtered multi-resolution volume rasterization.

Review: Problem and Objectives

- **Fundamental problem:**
  - Contain a subset of the input.
  - Exclude a subset of the complement.
  - Separate with an optimized boundary.
- **Fundamental objectives:**
  - Replace thin, sharp features with smooth, thick
  - Minimize boundary and simplify normal field
  - Produce output resembling the input
  - Handle input and complement symmetrically

Review: Novel Techniques

- **Published:**
  - **Mason:** Blend while matching input shape.
  - **Tightening:** Bound curvature, minimizing boundary.
- **Unpublished:**
  - **Tight hull:** Generalize constrained convex hull.
  - **Tight blend:** Simplify tight hull normal field.
  - **Medial cover:** Obtain topology from medial axis.

Development: Modeling

- Opening and closing
- Mason
- Tightening
- Tight blend
- Medial cover

↓ **SYMMETRY**

↓ **CURVATURE BOUND**

↓ **DEVELOPABILITY**

↓ **TOPOLOGY MANAGEMENT**

Development: Regularity

- Distance transform
- Regularity transform

↓

OPENING/CLOSING STABILITY

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Development: Convexity

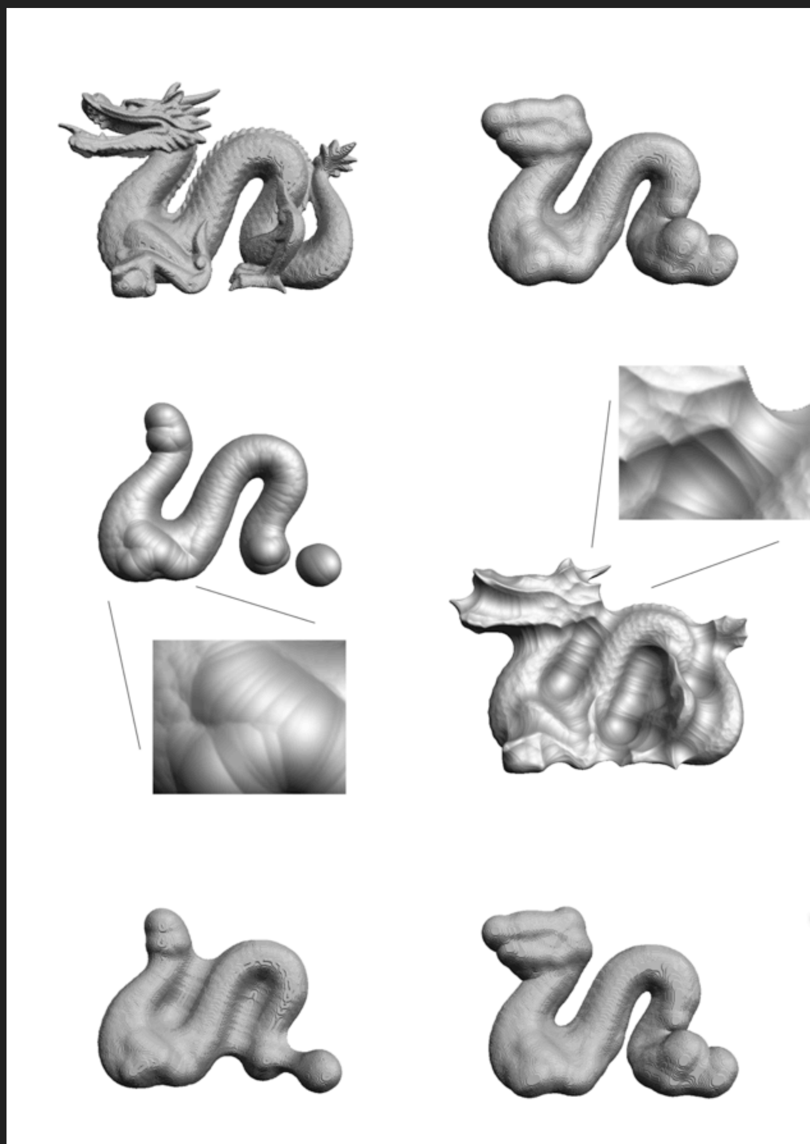
- Relative convex hull
- Tight hull

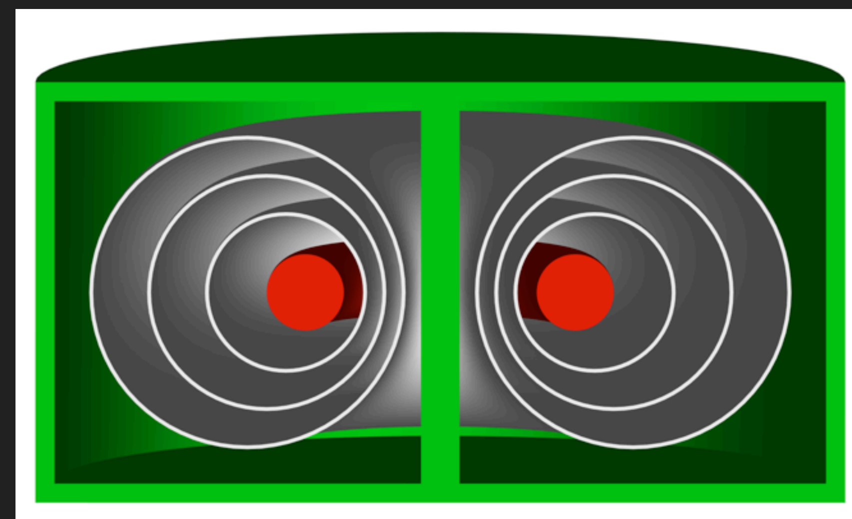
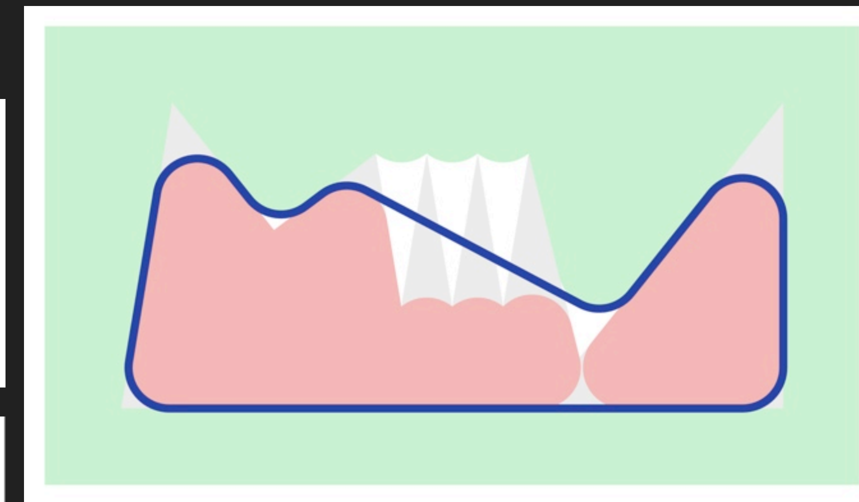
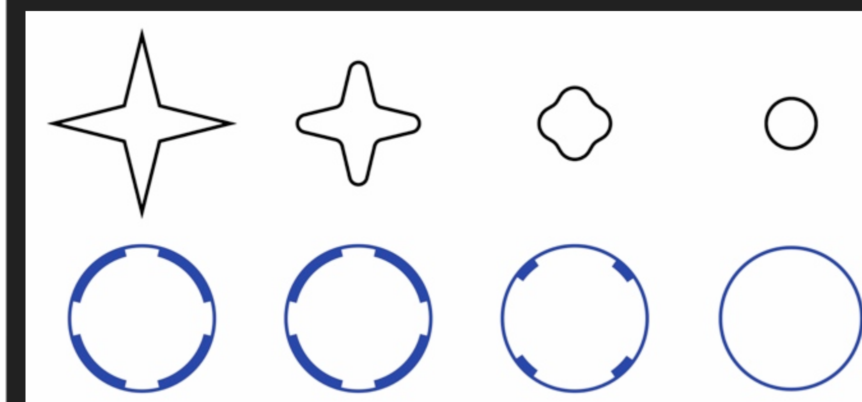
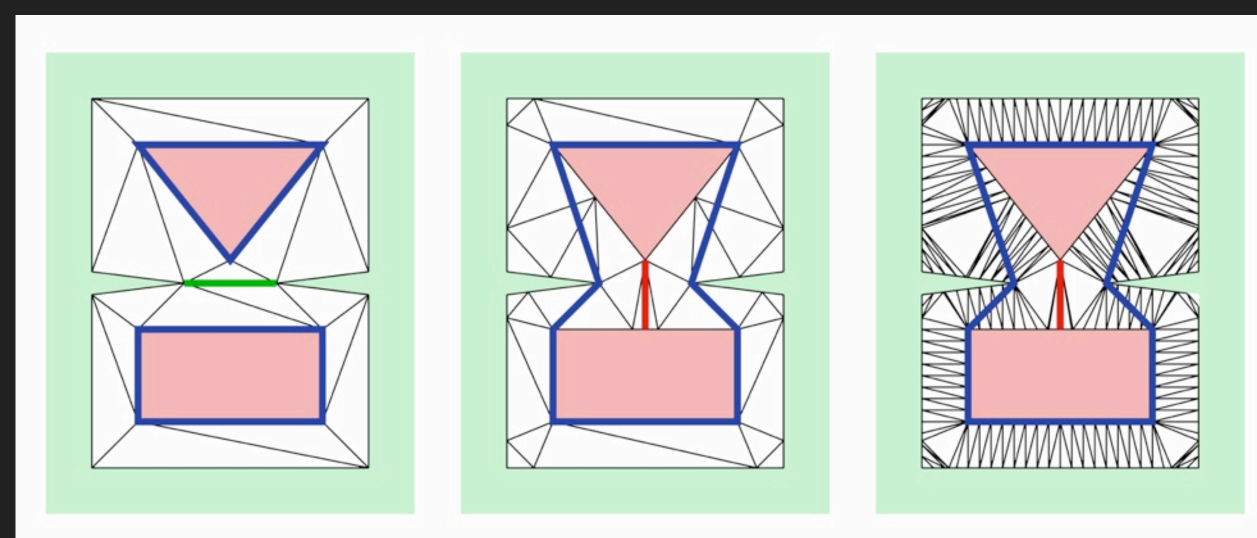
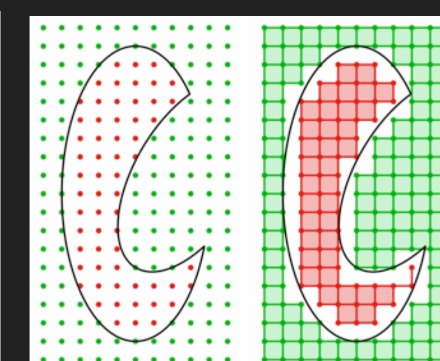
↓ **SYMMETRY**

OUR TECHNIQUES COMBINE MORPHOLOGICAL OPENING AND CLOSING, THE MORTAR, BOUNDARY MINIMIZATION, SLACK MINIMIZATION, AND MEDIAL AXIS TOPOLOGY MANAGEMENT TO ADDRESS IMPORTANT, LONGSTANDING PROBLEMS IN SOLID MODELING.

CONCLUSION

MASON, TIGHTENING, TIGHT HULL, TIGHT BLEND, MEDIAL COVER



TIGHTENING AND BLENDING

MASON, TIGHTENING, TIGHT HULL, TIGHT BLEND, MEDIAL COVER